MATH 1241 SPRING 2003
COMMON FINAL EXAMINATION
PART I

This exam is divided into three parts. You will have three hours for the entire exam, but you have only one hour to complete part I. No calculators are allowed during the first hour of the exam, hence **part I must be completed without the use of any calculator.** You may start working on parts II and III as soon as you finish part I, but you cannot use a calculator until your instructor collects part I at 9.00 a.m. At that point, you may use your calculator for the remainder of the exam. A special answer sheet is provided so that your answers can be machine graded.

These pages contain Part I which consists of 12 multiple choice questions. It has to be done without using any calculators.

- You must use a pencil with a soft black led (#2 or HB) to enter your answers on the answer sheet.
- For each question choose the response which best fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.
- There is no penalty for guessing.
- If you mark more than one answer to a question, that question will be scored as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.
- **Make sure that your name appears on the answer sheet and that you fill in the circles corresponding to your name.**

At the end of the examination you MUST hand in this test booklet, your answer sheet and all scratch paper.

Name: ______________________ Student ID. ______________________

Instructor: ______________________ Section No: ______________________

PART I

1. The value of the derivative of $g(x) = 4\pi^2 - 2e^x$ at $x = 2$ is

   a) $8\pi - 2e^2$ b) $4\pi^2 - 2e^2$ c) $-2e^2$ d) $-4e$ e) $8\pi - 2e^x$

2. If $f(x) = 3x^8 - 2x^3 + 5 + \frac{1}{x^2}$, then $f'(x)$ is

   a) $24x^7 - 6x^2 + \frac{1}{x^3}$ b) $24x^7 - 6x^2 - \frac{2}{x^3}$ c) $28x^7 - 3x^2 + 5 + \frac{1}{x}$

   d) $21x^7 - 4x^2 + \frac{1}{x}$ e) $24x^7 - 6x^2 + 5 + \frac{1}{x^3}$

3. If $h(x) = \ln(x+4)$, then its inverse $h^{-1}(x)$ is

   a) $1/\ln(x+4)$ b) $\ln(x-4)$ c) e^{x-4} d) $e^x - 4$ e) $1/(x+4)$

4. If $f(x) = \sqrt{2x^3+1}$, then $f'(x) =$

   a) $6x^2\sqrt{2x^3+1}$ b) $\frac{6x^2}{\sqrt{2x^3+1}}$ c) $\sqrt{6x^2+1}$

   d) $\frac{3x^2}{\sqrt{2x^3+1}}$ e) $\frac{1}{2\sqrt{2x^3+1}}$

5. The absolute maximum and absolute minimum values of $f(x) = x^4 - 2x^2 + 3$ on the interval $[-2, 3]$ are:

   a) 66, 2 b) 3, -2 c) 66, 11 d) 11, 3 e) 66, 0

6. If $g(\theta) = \cos(2\theta) + 3\tan\theta$, then $g'(\pi/3) =$

   a) $\sin(2\pi/3) + 3\cot(\pi/3)$ b) $2\sin(\pi/3) + 3\sec^2(\pi/3)$
   c) $-2\sin(2\pi/3) + 3\sec^2(\pi/3)$ d) $-\sin(2\pi/3)/2 + 3\cot(\pi/3)$
   e) $-2\sin(2\pi/3) + 3\csc^2(\pi/3)$

7. The derivative $h'(t)$ of the function $h(t) = \dfrac{-t+1}{2t^2+5t}$ is equal to

a) $\dfrac{-1}{4t+5}$ b) $\dfrac{2t^2-4t-5}{(2t^2+5t)^2}$ c) $\dfrac{-2t^2+4t+5}{(2t^2+5t)^2}$

d) $\dfrac{-6t^2-6t+5}{(2t^2+5t)^2}$ e) $\dfrac{-1}{(2t^2+5t)^2}$

8. If $h(x) = e^{x^2+\sin x}$, then its derivative $h'(x) =$

a) $(x^2+\sin x)\, e^{x^2+\sin x-1}$ b) $e^{2x+\cos x}$ c) $(2x+\cos x)\, e^{x^2+\sin x}$

d) $e^{x^2+\sin x}$ e) $\ln(x^2+\sin x)$

9. The derivative of the function $y = x\ln x$ is

a) $y' = 1+\ln x$ b) $y' = 1+\dfrac{1}{x}$ c) $y' = 1\cdot\dfrac{1}{x}$

d) $y' = \ln x$ e) $y' = \dfrac{\ln x}{x}$

10. $\lim\limits_{x\to 1}\left(\dfrac{\ln x}{1-x^3}\right)$ is equal to

a) $-1/3$ b) 0 c) $-1/2$ d) 1 e) does not exist

11. If $f(x) = \ln(x^2-3)$, then $f'(x) =$

a) $\dfrac{2x}{x^2-3}$ b) $\dfrac{2x-3}{x^2-3}$ c) $\dfrac{1}{x^2-3}$ d) $\ln(2x)$ e) $2x\ln(x^2-3)$

12. If $f'(x) = e^x - \cos x + 2x^3 + 5$, then the general antiderivative of $f'(x)$ is

a) $f(x) = e^x + \sin x + 6x + C$

b) $f(x) = e^x + \sin x + 2x^4 + 5x + C$

c) $f(x) = e^x - \sin x + x^4/2 + 5x + C$

d) $f(x) = e^x - \sin x + x^4 + 5 + C$

e) $f(x) = e^x - \sin x + 6x^2 + C$

MATH 1241 SPRING 2003
COMMON FINAL EXAMINATION
PART II

These pages contain Part II of the exam which consists of 13 multiple choice questions. You cannot use a calculator on any part of this exam until after 9.00 a.m., when part I will be collected. After that time, calculators are permitted on parts II and III of the exam. A special answer sheet is provided so that your answers can be machine graded.

- You must use a pencil with a soft black led (#2 or HB) to enter your answers on the answer sheet.
- For each question choose the response which best fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.
- There is no penalty for guessing.
- If you mark more than one answer to a question, that question will be scored as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.
- **Make sure that your name appears on the answer sheet and that you fill in the circles corresponding to your name.**

At the end of the examination you MUST hand in this test booklet, your answer sheet and all scratch paper.

Name: ______________________ Student ID. ____________________

Instructor: ____________________ Section No: ____________________

PART II

1. Let $f(x) = \begin{cases} \frac{\sqrt{x^2+4}-2}{x^2} & \text{if } x \neq 0 \\ -2 & \text{if } x = 0 \end{cases}$,

which of the following statements are true?

I. $\lim_{x\to 0} f(x) = -2$

II. f is discontinuous at 0

III. $\lim_{x\to 0} f(x) = \frac{1}{4}$

a) I only b) II only c) III only d) I and II only e) II and III only

2. The equation of the tangent line to the curve $y = 2x^3 - 4x$ at the point $(-1, 2)$ is

a) $y = x + 1$ b) $y = 2x + 1$ c) $y = 2x + 4$ d) $y = 6x - 12$ e) $y - 2 = x + 1$

3. Which of the given functions f and values of a are such that the derivative of f at a is given by the limit $\lim_{h\to 0} \frac{(2+h)^3 - 8}{h}$

a) $f(x) = 2 + x,\ a = 8$ b) $f(x) = (2+x)^3,\ a = 2$
c) $f(x) = 2,\ a = 8$ d) $f(x) = x^3,\ a = 8$
e) $f(x) = x^3,\ a = 2$

4. The slope of the tangent line to the curve $x^2y + 3xy^2 = 2x$ at the point $(1, -1)$ is equal to

a) $-1/2$ b) $-1/5$ c) -1 d) $1/2$ e) $-\sqrt{2}/5$

5. Suppose that $F = f \circ g$, where

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	−1	4
2	5	−4	−6	1

Then $F'(1)$ is equal to

a) 4 b) −24 c) 12 d) −2 e) −4

6. Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the radius is 25cm? (Note that $V = \frac{4}{3}\pi r^3$)

   a) $\frac{1}{25\pi}$cm/s b) 100πcm/s c) 25πcm/s d) $\frac{1}{100\pi}$cm/s e) $\sqrt[3]{\frac{300}{4\pi}}$cm/s

7. The function f satisfies the following conditions:

   $f'(3) = 0$ $f(3) = 2$ $f'(x) < 0$ if $0 < x < 3$ $f'(x) > 0$ if $x > 3$

   $f''(x) < 0$ if $0 \le x < 1$ or if $x > 4$ $f''(x) > 0$ if $1 < x < 4$

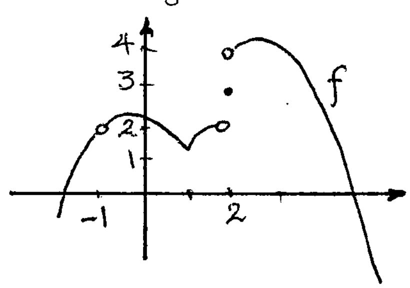
   Then the graph of f is

   a) concave down if $0 < x < 3$ and concave up if $x > 3$
   b) increasing if $1 < x < 4$ and decreasing if $0 \le x < 1$ or $x > 4$
   c) concave up if $1 < x < 4$ and has a local maximum at $x = 3$
   d) concave up if $1 < x < 4$ and has a local minimum at $x = 3$
   e) increasing if $1 < x < 4$ and has a point of inflection at $x = 3$

8. If a ball is thrown into the air with a velocity of 50ft /s, its height in feet after t seconds is given by $y = 50t - 16t^2$. The value of the velocity, $v(t)$, when $t = 3$ is

   a) $v(3) = 6$ ft/s b) $v(3) = 146$ ft/s
   c) $v(3) = 46$ ft/s d) $v(3) = -46$ ft/s
   e) $v(3) = 50$ ft/s

9. The graph of the function $f(x)$ is given below. Which of the following statements are true?



   I. $\lim_{x\to -1} f(x) = 2$ II. $\lim_{x\to 2} f(x) = 3$ III. $\lim_{x\to 2^+} f(x) = 4$

   a) I and III only b) I and II only
   c) II and III only d) I, II and III
   e) III only.

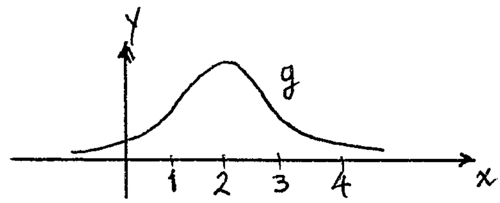
10. A function f is continuous on $(-\infty, \infty)$ and $f(1) = 2$, $f(2) = 3$, $f(3) = -4$, $f(4) = -5$. The Intermediate Value Theorem guarantees that the equation $f(x) = 0$ has a root on which of the following intervals?

a) $(1, 2)$ b) $(2, 3)$ c) $(3, 4)$ d) $(4, 5)$ e) $(0, 1)$

11. The domain and the range of the function $h(x) = \ln(2 - x) + 5$ are

a) $Domain(h) = [-10, 2]$ $Range(h) = [2.5, 7]$
b) $Domain(h) = (-\infty, 2]$ $Range(h) = [2.5, +\infty)$
c) $Domain(h) = (0, +\infty)$ $Range(h) = (-\infty, +\infty)$
d) $Domain(h) = (2, +\infty)$ $Range(h) = (5, +\infty)$
e) $Domain(h) = (-\infty, 2)$ $Range(h) = (-\infty, +\infty)$

12. The graph of the function g is given to the right. Which of the following values is the smallest?



a) $g'(0)$ b) $g'(1)$ c) $g'(2)$ d) $g'(3)$ e) $g'(4)$

13. Let $g(x) = e^x f(x)$, where $f(2) = 1$ and $f'(2) = -5$. Find $g'(2)$.

a) $6e^2$ b) $e^2 + 1$ c) $-5e^2$
d) $e^2 - 5$ e) $-4e^2$

MATH 1241
COMMON FINAL EXAMINATION
FREE RESPONSE SECTION
SPRING 2003

This exam is divided into three parts. These pages contain Part III which consists of 6 free response questions.

Please show all of your work on the problem sheet provided. We will not grade loose paper.

- If you are basing your answer on a graph on your calculator, sketch a picture of your graph on your sheet and be sure to label your window.
- **Make sure that your name appears on each page.**

At the end of the examination you MUST hand in this test booklet and all scratch paper.

PROBLEM	1	2	3	4	5	6
GRADE						

FREE RESPONSE SCORE: __________

Name: ______________________ Student ID No: ______________________

Instructor: ______________________ Section No: ______________________

PART III

1. A particle is moving along the x-axis, its position at time t is given by $x(t) = t^3 - 3t^2 + 4.5$, $t \geq 0$, where t is measured in seconds and x in meters.

(a) Find the velocity $v(t)$ at time t.

(b) When is the particle moving to the left?

(c) Find the total distance traveled during the first 5 seconds.

(d) Find the acceleration $a\,(t)$ at time t.

(e) When is the particle speeding up?

2. A box with a square base and open top must have a volume of $32,000\ cm^3$.

a) Find the surface area of the box (amount of material used) in terms of the base width x.

b) Find the dimensions of the box that minimizes the amount of material used.

3. The derivative of a function $f(x)$ is defined by $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

(a) Use this definition of the derivative to find $f'(x)$ if $f(x) = 2x^2 + 5$.

(b) Use the above definition of derivative to compute $f'(3)$ if $f(x) = \sqrt{x}$.

4. Complete the following steps to find the root of the equation $x^3 + 2x - 5 = 0$ using Newton's method.

$f(x) =$

$f'(x) =$

$x_{n+1} =$

take $x_1 = 1$, then

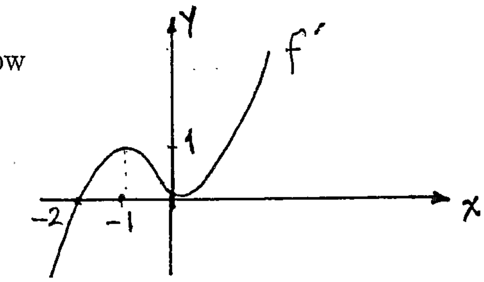
$x_2 =$

$x_3 =$

$x_4 =$

Thus, the root of the equation $x^3 + 2x - 5 = 0$ is $x =$

5. The graph of the derivative f' of a function f is shown below



(a) Determine the interval(s) over which f is decreasing.

(b) Determine the interval(s) over which f is concave up.

(c) Does f have a local maximum? If yes, give its x-coordinate.

(d) Does f have a local minimum? If yes, give its x-coordinate.

(e) Give the x-coordinate(s) of any point(s) of inflection.

6. A cubic polynomial function f is defined by $f(x) = 4x^3 + mx^2 + nx + 2$ where m and n are constants.

(a) Find $f'(x)$

(b) Find $f''(x)$

(c) Given that the function f has a local maximum at $x = -1$ and the graph has a point of inflection at $x = 2$. Find the values of m and n and give the formula for $f(x)$.