February 4, 2004 **Name** ____________________

The first nine problems count 6 points each and the final seven count as marked. There are 120 points available on this test.

Multiple choice section. Circle the correct choice(s). You do not need to show your work on these problems.

1. Which of the following is a factor of $x^4 - x$? Circle all those that apply.

   (A) x **(B)** $x-1$ **(C)** $x+1$ **(D)** x^2+x+1 **(E)** x^2-x+1

   Solution: The expression factors as follows: $x^4 - x = x(x^3-1) = x(x-1)(x^2+x+1)$, so A, B, and D are all factors.

2. How many roots does the equation below have?

$$x^2(x^2-3) - 4(x^2-3) = 0$$

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

   Solution: E. The expressions factors, so the equation can be written $x^2(x^2-3)-4(x^2-3) = (x-2)(x+2)(x-\sqrt{3})(x+\sqrt{3}) = 0$, so there are four roots.

3. $$\frac{1+\frac{1}{x}}{1-\frac{1}{x}} =$$

   (A) $\dfrac{x+1}{x-1}$ **(B)** $\dfrac{x-1}{x+1}$ **(C)** $x-1$ **(D)** $1-x$ **(E)** x

   Solution: A. Find a common denominator for numerator and denominator to get $(x+1)/x \div (x-1)/x = (x+1)/(x-1)$.

4. What is the radius of the circle whose equation is $x^2 - 8x + y^2 + 6y = 24$?

   (A) 4 **(B)** $\sqrt{24}$ **(C)** 5 **(D)** 6 **(E)** 7

   Solution: E. Complete the two squares to get $x^2-8x+16+y^2+6y+9 = 24+25$ which is the same as $(x-4)^2+(y+3)^2 = 7^2$, so the radius of the circle is 7.

5. Which of the following is a solution to $2(5-3x) - 2\cdot 5 - 3x = 108$? Circle all that apply.

   (A) -12 **(B)** -9 **(C)** -2 **(D)** 0 **(E)** none of these

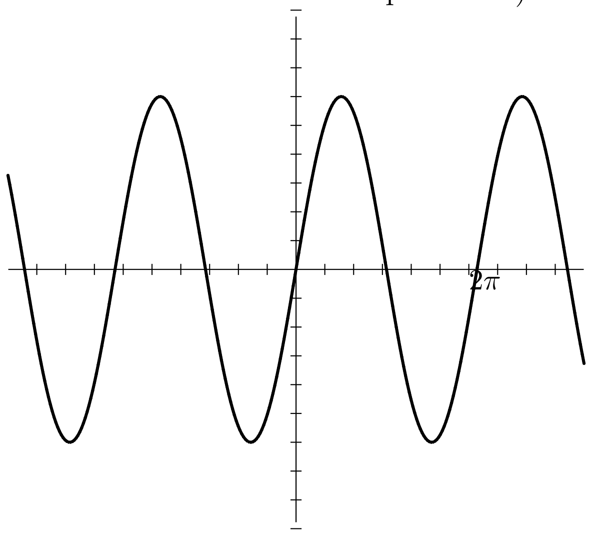
   Solution: A. The equation reduces to $-9x = 108$ so the only solution is $x = -12$.

6. Which of the following is a solution to $3(x-2)^3(x+1)^2 - 2(x-2)^2(x+1)^3 = 0$? Circle all that apply.

   (A) -2 **(B)** -1 **(C)** 0 **(D)** 2 **(E)** 8

   Solution: B.D.E. Again factoring is the key. The equation becomes $(x-2)^2(x+1)^2[3(x-2) - 2(x+1)] = 0$, so we get the three roots, $x = 2, x = -1$, and $x = 8$.

7. Consider the function $y = a\sin(bx)$, where a and b are constants, shown below. What is $a + b$? (Tick marks are located at unit positions.)



   (A) 2 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

   Solution: E. The amplitude is 6 and the period is 2π, so $a = 6$ and $b = 1$.

8. Suppose the functions f and g are given completely by the table of values shown.

x	$f(x)$	x	$g(x)$
0	2	0	5
1	7	1	7
2	5	2	4
3	1	3	2
4	3	4	6
5	6	5	3
6	0	6	1
7	4	7	0

What is $g^{-1}(f(3))$?

(A) 1 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

Solution: E. Note that $f(3) = 1$, so we want to find $g^{-1}(1)$ which is seen from the table to be 6.

9. Referring again to the two functions in the previous question, solve the equation $g(f(g(x))) = 5$ for x.

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

Solution: D. Note that $g(0) = 5$, so we need to solve $f(g(x)) = 0$. Note $f(6) = 0$, so we need to solve $g(x) = 6$, which we can see from the table is 4.

On all the following questions, **show your work.**

10. (7 points) Find the (implied) domain of the function $g(x) = \frac{\sqrt{x+1}}{x^2-9}$. Write your answer using interval notation.

    Solution: The number that must be excluded are those for which make the number in the square root negative and those which make the denominator zero. Thus, $x + 1 \geq 0$ and $x^2 - 9 \neq 0$. We can write this in interval notation as follows $[-1, 3) \cup (3, \infty)$.

11. (7 points) Let $f(x) = x^2 - x$. Compute in simplify $f(4), f(x + 1), f(x + h)$, and $\frac{f(x+h)-f(x)}{h}$, where $h \neq 0$.

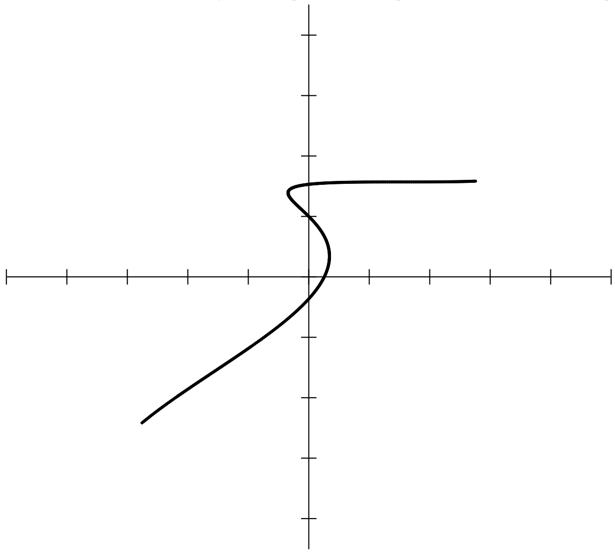
    Solution: $f(4) = 4^2 - 4 = 12$, $f(x+1) = (x+1)^2 - (x+1)$, and $f(x+h) = (x+h)^2 - (x+h)$. Finally, $\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-(x+h)-(x^2-x)}{h} = \frac{x^2+2xh+h^2-x-h-x^2+x}{h} = \frac{2xh+h^2-h}{h} = \frac{h(2x+h-1)}{h} = 2x + h - 1$.

12. (7 points) The slope of the line tangent to the graph of $f(x) = 2x^2 - x$ at the point $(1, 1)$ is 3. Find an equation for this tangent line.

    Solution: An equation is given in point-slope form by $y - 1 = 3(x - 1)$, which in slope-intercept form is $y = 3x - 2$.

13. (10 points) Sketch the curve below represented parametrically by $x = t - \sin(2t), y = t + \cos(t)$ for $-2 \leq t \leq 2$ on the grid provided.

    Solution: Use your graphing calculator to get.



14. (7 points) This of the computation of $J(x) = \sqrt{(x-2)^2+3}$ as a sequence of four simple computations. Find four functions, f, g, h, and k such that $J(x) = f \circ g \circ h \circ k(x)$.

    Solution: The operations in order are: subtract 2, square, add 3, take square root, so the functions are $k(x) = x - 2, h(x) = x^2, g(x) = x + 3, f(x) = \sqrt{x}$.

15. (15 points) A. Does the function $f(x) = 2x - 5$ have an inverse. If it does, find it. If not, state why it does not.

    Solution: Interchange the x and the y and solve for y to get $2y - 5 = x$ or $y = (x+5)/2$.

    B. Does the function $f(x) = \ln(2x - 5)$ have an inverse. If it does, find it. If not, state why it does not.

    Solution: Again, interchange the x and the y to get $x = \ln(2y - 5)$. Then solve for y by treating each side as an exponent. Thus $e^x = e^{\ln(2y-5)} = 2y - 5$. Therefore, $y = (e^x + 5)/2$.

Tear this page off your test booklet and take it home. Return it Friday at class time (or before) with a complete solution.

16. (20 points) A. First, complete the table below.

$$f(x) = \begin{cases} 2 & \text{if } x \leq -2 \\ |x| & \text{if } -2 < x \end{cases}$$

$$g(x) = \begin{cases} -x+3 & \text{if } x \leq 1 \\ 2x & \text{if } x > 1 \end{cases}$$

Solution:

x	$g(x)$	$f \circ g(x)$
-3	6	6
-2	5	5
-1	4	4
0	3	3
1	2	2
2	4	4
3	6	6
π	2π	2π

B. Find the composition $f \circ g$ of the two functions defined above. Remember that $f \circ g(x)$ is, by definition $f(g(x))$. Your final answer should not have the absolute value symbol in it.

Solution: First, compose all the part of g with those of f to get

$$f \circ g(x) = \begin{cases} 2 & \text{if } x \leq 1 \text{ and } -x+3 \leq -2 \\ |-x+3| & \text{if } x \leq 1 \text{ and } -x+3 > -2 \\ 2 & \text{if } x > 1 \text{ and } 2x \leq -2 \\ |2x| & \text{if } x > 1 \text{ and } 2x > -2 \end{cases}$$

Next solve each of the pairs of inequalities to eliminate some clauses and simplify others. The first clause disappears because the two inequalities $x \leq 1$ and $x \geq 5$ are incompatible. The second inequality becomes $x \leq 1$, the third also disappears, and the fourth becomes $x > 1$. Thus the function is

$$f \circ g(x) = \begin{cases} |-x+3| & \text{if } x \leq 1 \\ |2x| & \text{if } x > 1 \end{cases}$$

Actually, the absolute value symbols are not needed here because for all the values of x in the given ranges, the corresponding values in the absolute values is positive. So we can write the function as follows:

$$f \circ g(x) = \begin{cases} -x+3 & \text{if } x \leq 1 \\ 2x & \text{if } x > 1 \end{cases}$$

Please sign the following pledge: On my honor I declare that I have neither given nor received any help from anyone else on this test.

____