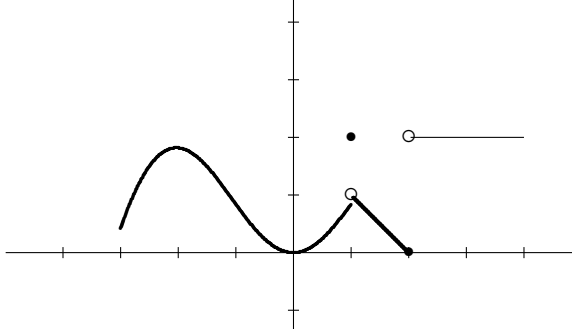


February 27, 2004

Name \_\_\_\_\_

There are 129 points available on this test. Each question is marked with its value. Multiple choice section. Circle the correct choice. You do not need to show your work on these problems.

1. (35 points) Questions (a) through (e) refer to the graph of the function  $f$  given below. Each tick mark represents one unit.



(a)  $\lim_{x \rightarrow 2^+} f(x) =$

- (A) 0    (B) 1    (C) 2    (D) 4    (E) does not exist

**Solution:** By the blotter test, the limit is 2.

(b)  $\lim_{x \rightarrow 2} f(x) =$

- (A) 0    (B) 1    (C) 2    (D) 4    (E) does not exist

**Solution:** By the blotter test, the limit does not exist.

- (c) A good estimate of  $f'(-2)$  is

- (A) -1    (B) 0    (C) 1    (D) 2    (E) there is no good estimate

**Solution:** The tangent line is close to horizontal, so 0 is a good estimate for  $f'(-2)$ .

- (d) A good estimate of  $f'(-1)$  is

- (A) -1    (B) 0    (C) 1    (D) 2    (E) there is no good estimate

**Solution:** The tangent line has negative slope at  $-1$ , so option A is the only real choice.

- (e) A good estimate of  $f'(3)$  is

- (A) -1    (B) 0    (C) 1    (D) 2    (E) there is no good estimate

**Solution:** The slope of the tangent line is zero because it is horizontal.

On all the following questions, **show your work.**

2. (15 points) Find the derivative of the function  $f(x) = 1/(x + 1)$  using the original definition of derivative.

**Solution:** We start as usual with the difference quotient.

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \left[ \frac{1}{x+h+1} - \frac{1}{x+1} \right] \div h \\ &= \left[ \frac{x+1}{(x+h+1)(x+1)} - \frac{x+h+1}{(x+h+1)(x+1)} \right] \div h \\ &= \left[ \frac{x+1 - (x+h+1)}{(x+h+1)(x+1)} \right] \div h \\ &= \frac{-h}{(x+h+1)(x+1)h} \\ &= \frac{-1}{(x+h+1)(x+1)} \end{aligned}$$

Next, take the limit as  $h \rightarrow 0$  to get  $f'(x) = -\frac{1}{(x+1)^2}$ .

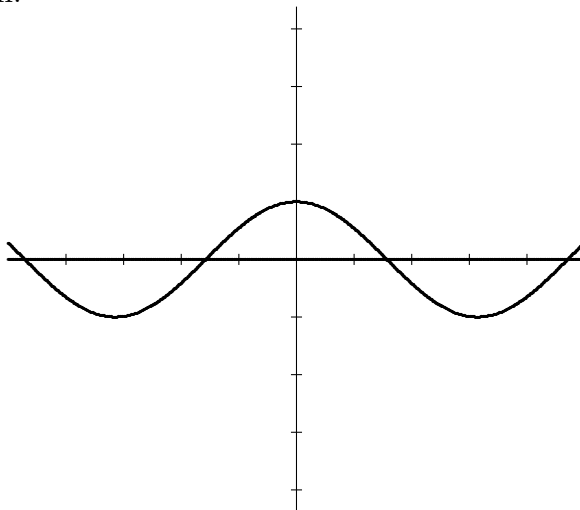
3. (15 points) Find the derivative of the function  $f(x) = \sqrt{x-2}$  using the original definition of derivative. Then use the information to find an equation for the line tangent to the graph of  $f$  at the point  $(6, 2)$

**Solution:** We start as usual with the difference quotient. We need to rationalize the numerator.

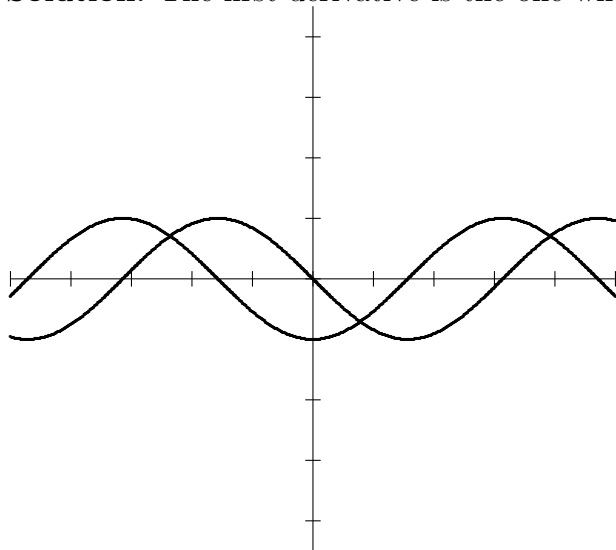
$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \left[ \sqrt{x+h-2} - \sqrt{x-2} \right] \div h \\ &= \frac{(\sqrt{x+h-2} - \sqrt{x-2})(\sqrt{x+h-2} + \sqrt{x-2})}{(\sqrt{x+h-2} + \sqrt{x-2})h} \\ &= \frac{x+h-2 - (x-2)}{(\sqrt{x+h-2} + \sqrt{x-2})h} \\ &= \frac{h}{(\sqrt{x+h-2} + \sqrt{x-2})h} \\ &= \frac{1}{(\sqrt{x+h-2} + \sqrt{x-2})} \end{aligned}$$

Next, take the limit as  $h \rightarrow 0$  to get  $f'(x) = \frac{1}{2\sqrt{x-2}}$ . To finish the problem, evaluate  $f'(x)$  at  $x = 6$  to get  $m = 1/4$ . The line passing through  $(6, 2)$  with slope  $1/4$  is given in point-slope form by  $y - 2 = (1/4)(x - 6)$  which we can rewrite as  $y = x/4 + 1/2$ .

4. (10 points) The graph of function  $f(x)$  on the interval  $[-5, 5]$  is given. On the same set of coordinate axes, sketch the graphs of  $f'(x)$  and  $f''(x)$ , being clear about which is which.



**Solution:** The first derivative is the one whose value at 0 is 0.



5. (15 points) Discuss the asymptotes of the function  $f$  defined by:

$$f(x) = \frac{(x+1)x^2(x^2-4)(2x-5)}{(x+2)x^3(4x^2-25)}$$

- A. Locate and describe all vertical asymptotes.

**Solution:** First reduce the fraction to lowest terms.

$$f(x) = \frac{(x+1)x^2(x-2)(x+2)(2x-5)}{(x+2)x^3(2x-5)(2x+5)} = \frac{(x+1)(x-2)}{x(2x+5)}.$$

Now we can read off the vertical asymptotes,  $x = 0$  and  $x = -5/2$ .

B. Locate all horizontal asymptotes.

**Solution:** To find the horizontal asymptotes, note that the degrees of the numerator and denominator are both 6, so we need only compute  $a_6/b_6 = 2/4 = 1/2$ , so the horizontal asymptote is the line  $y = 1/2$ .

6. (24 points) Find each limit below if it exists. If it does not exist, state why it does not. Explain how you arrived at your answer and of course, SHOW YOUR WORK

(a)  $\lim_{h \rightarrow 0} \frac{|h|}{h}$

**Solution:** The limit from the left is  $-1$  and the limit from the right is  $1$ , so the limit does not exist.

(b)  $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^3 - 1}$

**Solution:** Factor the denominator and cancel out the factor  $x - 1$  to get

$$\lim_{x \rightarrow 1} \frac{x + 1}{x^2 + x + 1} = 2/3.$$

(c)  $\lim_{x \rightarrow -2} \frac{4x + 8}{x^2 - 4}$

**Solution:** Rewrite the function as  $\frac{4(x+2)}{(x-2)(x+2)}$  and cancel the factor  $x + 2$  from both parts to get  $\frac{4}{x-2}$  whose limit as  $x \rightarrow -2$  is  $-1$ .

(d)  $\lim_{x \rightarrow \infty} \sqrt{x^2 + 6x - 3} - x$

**Solution:** Rationalize the expression to get

$$\begin{aligned} \sqrt{x^2 + 6x - 3} - x &= \frac{\sqrt{(x^2 + 6x - 3 - x)(x^2 + 6x - 3 + x)}}{\sqrt{x^2 + 6x - 3 + x}} \\ &= \frac{(x^2 + 6x - 3 - x^2)}{\sqrt{x^2 + 6x - 3 + x}} \\ &= \frac{(6x - 3)}{\sqrt{x^2 + 6x - 3 + x}} \\ &= \frac{(6x-3)}{\frac{x}{\sqrt{x^2+6x-3+x}}} \\ &= \frac{(6 - 3/x)}{\sqrt{x^2/x^2 + 6x/x^2 - 3/x^2 + 1}} \end{aligned}$$

which has limit  $3$  as  $x \rightarrow \infty$ .

7. (15 points) The position of a particle at time  $t$  is given by  $f(t) = 3t^2 + 5t + 3$ , where  $t$  is measured in seconds and  $f(t)$  is measured in feet.

(a) How far does the particle travel between the time  $t = 1$  and  $t = 4$ ?

**Solution:** The distance travelled is  $f(4) - f(1) = 3 \cdot 4^2 + 5 \cdot 4 + 3 - (3 \cdot 1^2 + 5 \cdot 1 + 3) = 71 - 11 = 60$ .

(b) What is the average speed in feet per second that the particle travels during this time interval  $[1, 4]$ ?

**Solution:** The average speed is the distance travelled divided by the time required,  $60 \div (4 - 1) = 20$  feet per second.

(c) What is the instantaneous speed of the particle when  $t = 4$ ?

**Solution:** Since  $f'(t) = 6t + 5$ , we get the instantaneous speed at  $t = 4$  by evaluating  $f'(4) = 29$ .