April 3, 1998 **Name** ______________________

In the first 4 problems each part counts 6 points each for a total of $6 \cdot 6 = 36$ points and the final 3 problems count as marked.

Multiple choice section. Circle the correct choice. You do not need to show your work on these problems.

1. Let $L(x)$ be the linearization of the function $f(x) = \sqrt{x+4}$ at the point $a = 0$. What is $L(.5)$?

   (A) 1.95 **(B)** 2.13 **(C)** 2.25 **(D)** 2.5 **(E)** 2.55

2. For how many points on the curve $x^2 + 2y^2 = 1$ does the tangent line have slope 1?

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** more than 4

3. The x-coordinate of one point on the curve $x^2 + 2y^2 = 4$ where the tangent line has slope -1 is (approximately, to three decimal places)

   (A) 1.234 **(B)** 1.347 **(C)** 1.546 **(D)** 1.633 **(E)** 1.668

4. Suppose the functions f and g are given completely by the table of values shown. The next four problems refer to the functions f and g given in the tables.

x	$f(x)$	$f'(x)$	x	$g(x)$	$g'(x)$
0	2	1	0	5	5
1	7	3	1	7	3
2	5	4	2	4	4
3	1	2	3	2	6
4	3	3	4	6	10
5	6	4	5	3	4
6	0	5	6	1	2
7	4	1	7	0	1

(a) The function h is defined by $h(x) = f(g(x))$. Use the chain rule to find $h'(2)$.

(A) 1 **(B)** 4 **(C)** 6 **(D)** 10 **(E)** 12

(b) The function k is defined by $k(x) = f(x) \cdot g(x)$. Use the product rule to find $k'(3)$.

(A) 1 **(B)** 4 **(C)** 6 **(D)** 10 **(E)** 12

(c) The function H is defined by $H(x) = f(x)/g(x)$. Use the quotient rule to find $H'(4)$.

(A) -12 **(B)** $-1/3$ **(C)** $-1/2$ **(D)** $3/10$ **(E)** 1

5. (25 points)

   (a) Find $\frac{d}{dx}(\tan x)$

   (b) Write an equation involving $\tan, \arctan$, the composition operation, and the identity function.

   (c) Differentiate both sides of the equation in (b).

   (d) Use the result in (c) to find an expression for $\frac{d}{dx}(\arctan x)$.

6. (20 points)

   (a) Find $\frac{dy}{dx}$ at the point $(-1, 1)$ if x and y are related by

   $$x^2 + xy - y^3 = xy^2$$

   (b) Use the information in (a) at find an equation for the line tangent to the curve

   $$x^2 + xy - y^3 = xy^2$$

   at the point $(-1, 1)$.

7. (28 points)

   (a) Let $V(t)$ be the volume of a cube with an edge of length $x(t)$. Find dV/dt in terms of dx/dt

   (b) Let $V(t)$ be the volume of a sphere with a radius $r(t)$. Find dV/dt in terms of dr/dt

   (c) Let $V(t)$ be the volume of a cube with surface area of $S(t)$. Find $V(t)$ as a function of $S(t)$. Find dV/dt in terms of dS/dt

   (d) Referring to part (c), suppose the surface area is changing at the rate $24in^2/sec$ precisely at the time when the volume is 1728 cubic inches. How fast is the volume changing at this moment?