MATH 1242
COMMON FINAL EXAMINATION
PART I
FALL 2003

This exam is divided into three parts. You have three hours for the entire test. Part I consists of 12 multiple choice questions to be done **without using any calculator.** Part I is collected after one hour (if exam has started at 8:00 a.m., Part I is collected at 9:00 a.m.). **Calculators are allowed only on Part II and III.** Part II contains 12 multiple choice questions. A special answer sheet is provided so that your answers can be machine graded. Part III contains 6 free response questions, for which your work on the exam will be graded.

These pages contain Part I which consists of 12 multiple choice questions. It must be done without using any calculator.

- You must use a pencil with a soft black lead (#2 or HB) to enter your answers on the answer sheet.
- For each question choose the response which best fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.
- There is no penalty for guessing.
- If you mark more than one answer to a question, that question will be scored as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.
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At the end of the examination you MUST hand in this test booklet, your answer sheet and all scratch paper.

Part I (MULTIPLE CHOICE, NO CALCULATORS).

1. Find $\int_1^2 x^3\,dx$.

(a) 0

(b) 4

(c) 7

(d) 9

(e) $\frac{15}{4}$

2. Find $\int_0^{\pi/2} \sin t\,dt$.

(a) 0

(b) -1

(c) 1

(d) 1/2

(e) $-1/2$

3. Find $\int x^2 \ln x\,dx$.

(a) $\frac{1}{3}x^2 + C$

(b) $\frac{1}{3}x^3 \ln x + C$

(c) $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$

(d) $2 + C$

(e) $\frac{1}{3}x^3 \ln x - \frac{1}{12}x^4 + C$

4. Find $\int 3e^{2x}\,dx$.

(a) $\frac{3}{2}e^{2x} + C$

(b) $6e^{2x} + C$

(c) $3e^{2x} + C$

(d) $3xe^{2x} + C$

(e) $3\ln(2x) + C$

5. Which of the following definite integrals gives the length of the curve $y = \sin x$ for $0 \le x \le \pi$?

(a) $\int_0^\pi \cos x\,dx$

(b) $\int_0^\pi \sqrt{1 - \cos x}\,dx$

(c) $\int_0^\pi \sqrt{1 + \cos x}\,dx$

(d) $\int_0^\pi \sqrt{1 + \cos^2 x}\,dx$

(e) $\int_0^\pi \sqrt{1 - \cos^2 x}\,dx$

6. Which of the following is the sum of the geometric series $\sum_{n=0}^{\infty} \left(-\frac{4}{5}\right)^n$?

(a) The series diverges.

(b) $-\frac{5}{9}$

(c) 5

(d) $\frac{9}{5}$

(e) $\frac{5}{9}$

7. Let $f(x) = \int_1^x \frac{1}{1+t^8}\,dt$. Then $f'(x) =$

(a) $\frac{1}{8x^7}$

(b) $\frac{1}{1+x^8}$

(c) $\frac{8x^7}{1+x^8}$

(d) $\int_1^x \frac{1}{8t^7}\,dt$

(e) $\ln|1+x^8| + C$

8. The Maclaurin series for $f(x) = \cos x$ is

(a) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

(b) $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

(c) $1 + x + x^2 + x^3 + x^4 + \cdots$

(d) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots$

(e) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

9. The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(a) converges by the ratio test.

(b) diverges by the ratio test.

(c) converges by comparison of its terms with the terms of the series $\sum_{n=1}^{\infty} \frac{1}{n}$.

(d) converges by the integral test.

(e) diverges by the integral test.

10. The following integral form appears in the Table of Integrals in your text:

$$\int \frac{\sqrt{a^2-u^2}}{u^2}\,du = -\frac{1}{u}\sqrt{a^2-u^2} - \sin^{-1}\frac{u}{a} + C$$

Using this form and the substitution $u = x^2$ we obtain $\int \frac{\sqrt{1-x^4}}{x^4}\cdot 2x\,dx =$

(a) $-\frac{1}{x}\sqrt{1-x^2} - \sin^{-1}x + C$

(b) $-\frac{1}{x}\sqrt{1-x^4} - \sin^{-1}x + C$

(c) $-\frac{1}{x^2}\sqrt{1-x^2} - \sin^{-1}x^2 + C$

(d) $-\frac{1}{x^2}\sqrt{1-x^4} - \sin^{-1}x^2 + C$

(e) $-\frac{1}{x^2}\sqrt{1-x^2} - \sin^{-1}x + C$

11. The average value of $f(x) = \frac{1}{x}$ on $[1, 4]$ is

(a) 1

(b) 3

(c) $\ln 4$

(d) $\frac{1}{2}\ln 4$

(e) $\frac{1}{3}\ln 4$

12. (This problem relates to Hooke's Law, which states that the force required to maintain a spring stretched x units beyond its natural length is proportional to x: $f(x) = kx$, where k is a positive constant.) Suppose that a force of 10 N is required to hold a spring stretched from its natural length of 0.3 m to a length of 0.5 m. How much work is required to stretch it from its natural length of 0.3 m to a length of 0.5 m?

(a) 1 J

(b) 2 J

(c) 3 J

(d) 4 J

(e) 5 J

MATH 1242 FALL 2003

COMMON FINAL EXAMINATION

PART II

Name:____________________ Instructor:____________________

Student ID #:____________________ Section/Time:____________________

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Part II (MULTIPLE CHOICE, CALCULATORS ALLOWED).

1. A table of values of a function f is shown. We wish to estimate $\int_0^6 f(x)\,dx$. Do this by finding the Riemann sum for f on $[0,6]$, using three equal subintervals and taking the sample points to be the right endpoints.

x	0	2	4	6
$f(x)$	5	1	2	1

(a) 0

(b) 2

(c) 4

(d) 6

(e) 8

2. Find the value of the following improper integral (if it converges): $\int_1^\infty \frac{1}{x^{5/4}}\,dx$.

(a) 0

(b) $-\frac{9}{4}$

(c) 4

(d) $\frac{1}{4}$

(e) The integral diverges.

3. Find the area of the finite region bounded by the curves $y = x^2$ and $y = 1$.

(a) 0

(b) $\frac{2}{3}$

(c) π

(d) $\frac{3}{2}$

(e) $\frac{4}{3}$

4. Which of the following series diverges?

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

(c) $\sum_{n=1}^{\infty} 1^n$

(d) $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$

(e) $\sum_{n=1}^{\infty} \frac{1}{e^n}$

5. A region in the plane is bounded by the curves $y = x^3$, $y = 0$, and $x = 2$. Find the volume obtained when this region is rotated about the x-axis.

(a) 8π

(b) 8

(c) $\frac{128\pi}{7}$

(d) 4π

(e) $\frac{2\pi\sqrt[3]{4}}{2}$

6. Find the interval of convergence of the following power series: $\sum_{n=1}^{\infty} \frac{(x-2)^n}{2^n}$.

(a) $(-0, 1)$

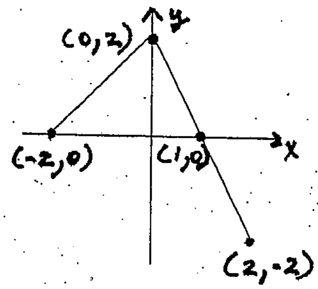
(b) $(0, 4)$

(c) $(-\infty, \infty)$

(d) $(-1, 1)$

(e) $(-2, 2)$

7. Consider the graph of the function f at right.



Find $\int_{-2}^{2} f(x)\,dx$.

(a) 4

(b) 3

(c) 2

(d) 1

(e) 0

8. The Taylor expansion of $f(x) = x^3 - 5x + 4$ about $a = 1$ is

(a) $-2(x-1) + 3(x-1)^2 + (x-1)^3$

(b) $(x-1) + (x-1)^2 + (x-1)^3$

(c) $1 + (x-1) + \frac{(x-1)^2}{2!} + \frac{(x-1)^3}{3!}$

(d) $1 + (x-1) + \frac{(x-1)^2}{2!} + \frac{(x-1)^3}{3!} + \frac{(x-1)^4}{4!} + \cdots$

(e) 0

9. The finite region in the plane bounded by the curves $y = x^2$ and $y = 1$ is rotated about the line $x = 1$ to form a solid. The volume of this solid is

(a) 2π

(b) $\frac{8\pi}{3}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{6}$

(e) $\frac{17\pi}{6}$

10. Consider the following integral: $\int_{-1}^{2} \frac{1}{x^2}\,dx$. Which of the following statements is correct?

(a) $\int_{-1}^{2} \frac{1}{x^2}\,dx = -\frac{3}{2}$

(b) $\int_{-1}^{2} \frac{1}{x^2}\,dx = \frac{3}{2}$

(c) $\int_{-1}^{2} \frac{1}{x^2}\,dx = \ln 4$

(d) $\int_{-1}^{2} \frac{1}{x^2}\,dx = -\frac{1}{2}$

(e) $\int_{-1}^{2} \frac{1}{x^2}\,dx$ is a divergent improper integral.

11. Consider the sequence with nth term $a_n = \frac{n^2}{e^n}$. Which of the following is correct?

(a) The sequence diverges.

(b) $\lim_{n\to\infty} a_n = 1$.

(c) $\lim_{n\to\infty} a_n = \frac{\infty}{\infty}$.

(d) $\lim_{n\to\infty} a_n = 0$.

(e) $\lim_{n\to\infty} a_n = -1$.

12. Let f be a function such that $f(0) = 2$, $f(1) = 4$, and $\int_0^1 f(x)\,dx = 3$. Using integration by parts, we find that $\int_0^1 x f'(x)\,dx =$

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

MATH 1242
COMMON FINAL EXAMINATION
FREE RESPONSE SECTION
FALL 2003

This exam is divided into three parts. These pages contain Part III which consists of 6 free response questions.

Please show all of your work on the problem sheet provided. We will not grade loose paper.

- If you are basing your answer on a graph on your calculator, sketch a picture of your graph on your sheet and be sure to label your window.
- **Make sure that your name appears on each page.**

At the end of the examination you MUST hand in this test booklet and all scratch paper.

PROBLEM	1	2	3	4	5	6
GRADE						

FREE RESPONSE SCOPE: __________

Name: ____________________ Student No: ____________________

Instructor: ____________________ Section No: ____________________

Part III (FREE RESPONSE, CALCULATORS ALLOWED). Note: You must show your work in order to receive credit.

1. (a) (2 points) Write down (either from memory or by deriving it) the Maclaurin power series expansion for $f(x) = e^x$, valid for all real x.

(b) (2 points) Use (a) and substitution to obtain a power series expansion for e^{-x^2}.

(c) (3 points) Use (b) to obtain a series expansion for $\int_0^{1/2} e^{-x^2}\,dx$.

(d) (3 points) Use (c) to estimate $\int_0^{1/2} e^{-x^2}\,dx$ with error less than 0.01.

2. (a) (4 points) Find $\int \frac{x^2}{\sqrt{2x^3+1}}\,dx$.

(b) (3 points) Find $\int \frac{x+1}{x^2-4}\,dx$.

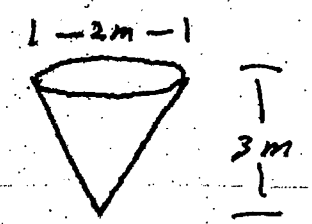
(c) (3 points) Find $\int \frac{x}{x^2+4}\,dx$.

3. Test each of the following series for convergence, and state your conclusion. (Each part counts 5 points.) You must show all your work and what convergence test you are using in each part.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{n}{n^3+1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

4. (10 points) A right circular conical tank (pictured below) is partially filled with water to a depth of 1 meter. Find the work done in pumping all the water out at the top of the tank. (The acceleration due to gravity is 9.8 m/s^2, and the density of water is 1000 kg/m^3.) To obtain full credit, you must show the details of your calculations.



5. Consider the region in the plane bounded by the curves $y = 0$, $y = e^x$, $x = 0$, and $x = 1$.

(a) (4 points) Find the area of the region.

(b) (3 points) Find the y-coordinate of the centroid of the region.

(c) (3 points) Find the x-coordinate of the centroid of the region.

6. (a) (5 points) Use Simpson's rule with $n = 6$ subintervals to approximate $\int_1^4 \frac{1}{x}\,dx$.

(b) (5 points) The error estimate when Simpson's rule is used to approximate $\int_a^b f(x)\,dx$ is given by

$$|E_n| \leq K\frac{(b-a)^5}{180n^4},$$

where n is the (even) number of subintervals and K is an upper bound for $|f^{(4)}(x)|$ on $[a, b]$. (Recall that $f^{(4)}$ is the fourth derivative of f.) With this estimate, determine the minimum value of n necessary to approximate $\int_1^4 \frac{1}{x}\,dx$ to within 0.001.