MATH 1242 SPRING 2004
COMMON FINAL EXAMINATION
PART I

Name_______________________________ Instructor:______________________
Student ID #__________________________ Section/Time____________________

This exam is divided into three parts. **Calculators are not allowed on Part I**. You have three hours for the entire test, but you have only one hour to finish Part I. You must turn in this test booklet for Part I at 9:00 am. You may start working on the other two parts of the exam whenever you are done with Part I, but you cannot use your calculator until ALL of the Part I questions are collected. After these questions are collected, your instructor will announce that calculators are allowed on Parts II and III.

These pages contain Part I, which consists of 13 multiple-choice questions. These questions must be answered without the use of a calculator.

- You must use a pencil with a soft black lead (#2 or HB) to enter your answers on the answer sheet.
- For each question, choose the response which best fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.
- There is no penalty for guessing.
- If you mark more than one answer to a question, the question will be scored as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.
- **Make sure that your name appears on the answer sheet and that you fill in the circles corresponding to your name.**

At approximately 9 am, you MUST hand in this test booklet. At the end of the exam you MUST hand in all remaining test materials including test booklets, answer sheet, and scratch paper.

Part I (MULTIPLE CHOICE, CALCULATORS NOT ALLOWED).

1. $\int (8x^2 + 2x - 1)dx =$

(a) $2x^4 + x^3 - x^2 + C$

(b) $4x^3 + 3x^2 - x + C$

(c) $4x^3 - x + C$

(d) $\frac{8}{3}x^3 - x^2 + x + C$

(e) $\frac{8}{3}x^3 + x^2 - x + C$

2. $\int_0^{\pi/2} \sin^2(x)\cos(x)dx$

(a) -1

(b) $-1/2$

(c) $1/3$

(d) $-1/3$

(e) 0

3. $\int_1^3 (2x + 3x^2)dx =$

(a) 35 (b) 8 (c) 34 (d) 94 (e) 36

4. If $\int_0^4 f(x)dx = 7 \, and \int_2^4 f(x)dx = 5, then \int_0^2 f(x)dx =$

(a) 8 (b) -8 (c) 2 (d) -2 (e) 4

(CALCULATORS NOT ALLOWED)

5. Let f(x) = x^2 on the interval [0, 2]. Let the interval be divided into two equal subintervals of widths Δx_1 and Δx_2. Find the value of the Riemann sum $\sum_{i=1}^{2} f(x_i^*)\Delta x_i$ if each x_i^* is the midpoint of its subinterval.

(a) $5/6$ (b) $7/6$ (c) $5/2$

(d) $7/3$ (e) $7/4$

6. If $F(x) = \int_0^x (t^3+1)^{1/2}$, then $F'(2) =$

(a) –3 (b) –2 (c) 2 (d) 3 (e) 18

7. Determine the limit as $n \to \infty$ of the sequence $a_n = \frac{(-1)^n}{\sqrt{n}}$

(a) –1 (b) 0 (c) ½ (d) 1 (e) $\sqrt{2}$

8. $\int_0^1 \frac{x^2}{(x^3+1)^2} dx =$

(a) $\frac{3}{4}$ (b) 2 (c) $\frac{3}{7}$ (d) $\frac{7}{3}$ (e) $\frac{1}{6}$

9. If $\int_3^a 3x^2 dx = 37$, then $a =$

(a) 4 (b) 5 (c) 6 (d) 7 (e) 8

10. The improper integral $\int_0^1 x^{-1/2} dx =$

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 1 (d) 2 (e) is divergent

(CALCULATORS NOT ALLOWED)

11. $\int_0^1 \frac{4}{2x+1}dx$

(a) $\frac{1}{2}\ln(3)$

(b) $2\ln(3)$

(c) $\ln(3)$

(d) e^3

(e) $2e^3$

12. Evaluate the improper integral $\int_0^\infty xe^{-x^2}dx$.

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) e (e) e^{-1}

13. Find $\int x^2 \ln x dx$.

(a) $\frac{x^2}{3}+C$

(b) $\frac{x^3}{3}(x\ln x - x)+C$

(c) $\frac{x^3}{3}\ln x+\frac{x^3}{9}+C$

(d) $\frac{1}{\ln x}+C$

(e) $\frac{x^3}{3}\ln x-\frac{x^3}{9}+C$

END OF PART I

CALCULATOR USE IS NOT PERMITTED UNTIL THE ANSWER SHEETS ARE COLLECTED FOR PART I. You may work on other parts of the exam before Part I answer sheets are collected, but without using a calculator.

MATH 1242 SPRING 2004
COMMON FINAL EXAMINATION
PART II

These pages contain Part II which consists of 12 multiple-choice questions. After the test booklets for Part I have been collected, and your instructor announces that calculators are OK, you are allowed to use a calculator on this part of the exam.

- You must use a pencil with a soft black lead (#2 or HB) to enter your answers on the answer sheet.
- For each question, choose the response which best fits the question.
- If you wish to change an answer, make sure that you compoletely erase your old answer and any other extraneous marks.
- There is no penalty for guessing.
- If you mark more than one answer to a question, the question will be scored as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.
- **Make sure that your name appears on the answer sheet and that you fill in the circles corresponding to your name.**

At the end of the exam, you MUST hand in all remaining test materials including test booklets, answer sheet, and scratch paper.

Part II (MULTIPLE CHOICE, CALCULATORS ALLOWED).

14. The area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$ is

(a) $\frac{2}{3}$ (b) $\frac{8}{3}$ (c) 4 (d) $\frac{14}{3}$ (e) $\frac{16}{3}$

15. A region in the plane is bounded by the curves $y = x^2$, $y = 0$, and $x = 2$. Find the volume obtained when this region is rotated about the x-axis.

(a) 32π (b) 16π (c) $32\pi/5$ (d) 4π (e) 16

16. The geometric series $\sum_{n=1}^{\infty} \frac{2^n}{3^{n-1}}$

(a) converges to 2 (b) converges to $\frac{2}{3}$ (c) converges to 6

(d) diverges (e) converges, but cannot be calculated explicitly

17. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$

(a) is divergent.

(b) is absolutely convergent.

(c) is convergent, but not absolutely convergent.

(d) equals 0.

(e) has only a finite number of terms.

18. The series $\sum_{n=1}^{\infty} \frac{1}{n^{2-\alpha}}$ converges if and only if

(a) $\alpha < 1$ (b) $-1 < \alpha < 1$ (c) $\alpha \le 1$ (d) $\alpha > 1$

(e) $\alpha \ge 1$

19. As $n \to \infty$, the sequence $\left\{ \frac{\ln(n^2)}{n} \right\}$

(a) converges to 0
(b) converges to 1
(c) diverges to ∞
(d) converges to $\frac{1}{n}$
(e) oscillates and never converges.

20. The interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 3^n}$ is

(a) $-1<x<5$

(b) $-1 \le x < 5$

(c) $-1 < x \le 5$

(d) $-1 \le x \le 5$

(e) $-1 < x < 0$

21. The values of A and B in the partial fraction decomposition of $\frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$ are

(a) A = 1, B = 1

(b) A = -1, B = 1

(c) A= 1, B = -1

(d) A = -1, B = -1

(e) A = 1, B = 0

22. Find the arclength of the curve x = 2t, $y = 2t^{3/2}$, $0 \le t \le 4$

(a) 15.2432
(b) 16.0011
(c) 27.326
(d) 18.1468
(e) 33.3333

23. One of the formulas from a table of integrals is $\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$. Use this to find $\int \frac{4}{4+x^2}\,dx$.

(a) $2\tan^{-1}\left(\frac{x}{2}\right) + C$

(b) $\frac{1}{2}\tan^{-1}(2x) + C$

(c) $2\ln(4 + x^2) + C$

(d) $4\ln(4 + x) + C$

(e) $4\ln(4 + x^2) + C$

24. The Maclaurin series for f(x) = sin x is

(a) $1 + x + x^2 + x^3 + \cdots$

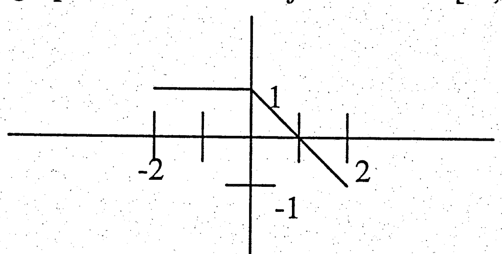
(b) $1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

(c) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

(d) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

(e) $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$

25. Consider the graph of the function *f* defined on [-2, 2]:



Find $\int_{-2}^{2} f(x)\,dx$.

(a) 3 (b) 2.5 (c) 2 (d) 1.5 (e) 0

MATH 1242
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PART III

SPRING 2004

Name______________________________ Instructor:_______________
Student ID #_________________________ Section/Time_____________

These pages contain Part III, which consists of 6 free-response questions.

Please show all your work in this test booklet. Loose paper will not be graded.

- **If you are basing your answer on a graph on you calculator, sketch this graph in the answer booklet. Be sure to label your window by putting a scale on the axes.**

- **Make sure that your name appears on each page.**

At the end of the exam, you MUST hand in all remaining test materials including test booklets, answer sheet, and scratch paper.

PROBLEM	**1**	**2**	**3**	**4**	**5**	**6**
GRADE						

FREE RESPONSE SCORE:_______________

Part III (FREE RESPONSE, CALCULATORS ALLOWED). You must show your work in order to receive credit.

1. Consider the finite region in the plane bounded by the curves $y = x^2$ and $y = 1$.

(a) (5 points) Set up an integral to find the volume generated by revolving the area about the x-axis. (Evaluation of the integral is not required.)

(b) (5 points) Set up an integral to find the volume generated by revolving the area about the line $x = 1$. (Evaluation of the integral is not required.)

2. Which of the following series converge? State which test you use to determine convergence or divergence.

(a) (5 points) $\sum_{n=1}^{\infty} \frac{n^2}{3n^2+2}$

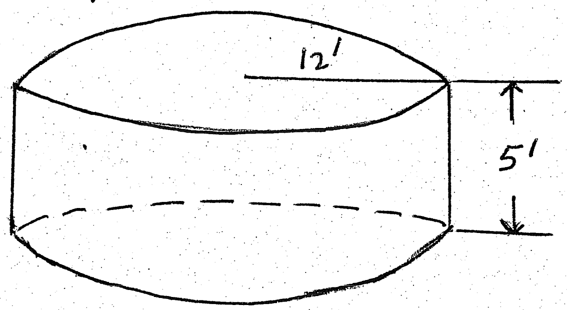
(b) (5 points) $\sum_{n=1}^{\infty} \frac{2}{n^3+5}$

3. Let $f(x) = \ln(x)$.

(a) (6 points) Write the third-degree Taylor polynomial approximation for $f(x)$ about $x = 1$.

(b) (4 points) Use the Taylor polynomial in (a) to approximate $\ln(2)$.

4. A circular swimming pool has a diameter of 24 feet, a height of 5 feet, and the depth of the water in the pool is 4 feet. How much work is required to pump all the water out over the side? (Use the fact that water weighs 62.5 lb/ft^3.)



5. (a) (4 points) Use Simpson's rule with n = 4,

$$S_4 = \frac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+f(x_4)],$$

to approximate $\ln 2 = \int_1^2 \frac{1}{x}\,dx$.

(b) (4 points) The error estimate for Simpson's rule is given by $|E_n| \le K\frac{(b-a)^5}{180n^4}$, where K is an upper bound for the fourth derivative of f. Estimate the error of the numerical integration in part (a) above.

(c) (2 points) What is the smallest value of n that will guarantee that the estimate by Simpson's rule will be accurate to within 0.001?

6. Consider the finite region bounded by the curves $x = y^2$ and $x = 2$.

(a) (6 points) Find the area of the region.

(b) (4 points) Find the centroid of the region.