September 27, 2004 **Name** ______________________________

On all the following questions, **show your work.** There are 145 points available on this test. Do not try to do all the problems. Try to find four or five that you you can do well.

1. (15 points) Let $f(x) = 1/x$ for all $x > 0$, and let $[a, b] = [1, 3]$.

   (a) Let $n = 4$ and use left endpoints for sample points to find the approximating sum. That is, compute L_4.

   (b) Find the n^{th} approximating sum, also using left endpoints. In other words, find an expression for L_n. You need not evaluate the limit as $n \to \infty$.

2. (30 points) Use the evaluation theorem, etc. to find each of the definite integrals below.

   (a) $\displaystyle\int_0^{\ln 3} 2e^{2x} dx$

   (b) $\displaystyle\int_1^6 \frac{x^2 - 3x + 5}{x^2}\, dx$

   (c) $\displaystyle\int_0^{\pi/4} \sec^2 x\, dx$

   (d) $\displaystyle\int_0^1 \frac{1}{\sqrt{1 - x^2}}\, dx$

   (e) $\displaystyle\int_0^1 \frac{x}{1 + x^4}\, dx$

3. (15 points) Evaluate $\displaystyle\int_0^\infty 1/x^2\, dx$.

4. (20 points) Let $g(x) = \displaystyle\int_0^x t^2 - 6t + 5\, dt$.

   (a) Over what intervals is g increasing?

   (b) Over what intervals is g concave upwards?

   (c) What is the maximum value of g over the interval $[0, 8]$?

5. (20 points) Suppose f is defined by:

$$f(x) = \begin{cases} 0 & \text{if } 0 \le x \le 1 \\ x-1 & \text{if } 1 < x \le 2 \\ 3-x & \text{if } 2 < x \le 3 \\ 0 & \text{if } 3 < x \end{cases}$$

Let $g(x) = \int_0^x f(t)dt$.

(a) Find an expression for $g(x)$ similar to the one for $f(x)$.

(b) Sketch the graphs of g and f.

(c) Compute $g'(x)$.

6. (15 points) Evaluate $\int_0^1 \ln x \; dx$.

7. (15 points) Let $f(x) = \displaystyle\int_0^{x^2} \frac{t}{t^3 - 2}\, dt$. Then $f'(x) = \frac{2x^3}{x^6 - 2}$. Explain why this is the case. How does the chain rule play a part here? What functions are being composed?

8. (15 points) Consider the integral $\displaystyle\int_{-2}^{3} 1/x \, dx$.

   (a) It is tempting to evaluate this integral by antidifferentiating $f(x) = 1/x$, getting $F(x) = \ln|x|$, and then to measuring the growth of $F(x)$ over the interval $[-2, 3]$ to get $\ln|3| - \ln|-2| = \ln 3 - \ln 2 = \ln(3/2)$. Explain why this is wrong.

   (b) Is there are reasonable approach to this problem?