February 18, 2005 **Name** _____

On all the following questions, **show your work.** There are 130 points available on this test. Do not try to do all the problems. Try to find four or five that you you can do well.

1. (15 points) You are given the four points in the plane $A = (0, 0), B(2, 5), C = (8, -1)$ and $D = (12, 1)$. The graph of the function f consists of the three line segments AB, BC and CD. Find the integral by interpreting the integral in terms of sums and/or differences of areas of elementary figures.

2. (15 points) Let $f(x) = 1/x$ for all $x > 0$, and let $[a, b] = [1, 3]$.

   (a) Let $n = 4$ and use right endpoints for sample points to find the approximating sum. That is, compute R_4.

   (b) Find the n^{th} approximating sum, also using right endpoints. In other words, find an expression for R_n. You need not evaluate the limit as $n \to \infty$.

3. (20 points) Find the following indefinite integrals.

   (a) $\int \frac{2x}{x^2+4} dx$

   (b) $\int \frac{1}{x^2+4}\, dx$

   (c) $\int \cos^3 x \sin^2 x\, dx$

4. (20 points) Let $g(x) = \int_0^x (4-t)(2+t)(5+t)\, dt$. Over what intervals is g increasing?

5. (30 points) Use the evaluation theorem as needed to find each of the definite and improper integrals below.

   (a) $\displaystyle\int_3^4 \frac{x+1}{x^2-4}\,dx$

   (b) $\displaystyle\int_0^2 xe^{x^2}\,dx$

   (c) $\displaystyle\int_0^1 x^2(x-1)^8\,dx$

   (d) $\displaystyle\int_e^\infty (x\ln x)^{-1}\,dx$

   (e) $\displaystyle\int_1^\infty 1/x^2\,dx.$

6. (15 points) Let $f(x) = \displaystyle\int_0^{\sqrt{x}} \frac{t^2}{t^4+2}\,dt$. Then $f'(x) = \frac{\sqrt{x}}{2x^2+4}$. Explain why this is the case. How does the chain rule play a part here? What functions are being composed?

7. (15 points) Consider the integral $\displaystyle\int_{-2}^{3} 1/x\,dx$.

   (a) It is tempting to evaluate this integral by antidifferentiating $f(x) = 1/x$, getting $F(x) = \ln|x|$, and then to measuring the growth of $F(x)$ over the interval $[-2, 3]$ to get $\ln|3| - \ln|-2| = \ln 3 - \ln 2 = \ln(3/2)$. Explain why this is wrong.

   (b) Is there are reasonable approach to this problem?