February 23, 2006 **Name** _____

On all the following questions, **show your work.** There are 143 points available on this test. Do not try to do all the problems. Try to find four or five that you you can do well.

1. (10 points) Suppose $\int_{-8}^{-5} f(x)dx = 2$, $\int_{-8}^{-7} f(x)dx = 10$, $\int_{-6}^{-5} f(x)dx = 8$.

   (a) Find $\int_{-7}^{-6} f(x)dx =$

   (b) $\int_{-6}^{-7} (2f(x) - 10)dx =$

2. (15 points) Given
$$f(x) = \int_0^x \frac{t^2 - 4}{1 + \cos^2(t)} dt$$
At what value of x does the local max of $f(x)$ occur?

3. (24 points) Find the following indefinite integrals.

(a) $\int \cos^3\theta \sin^3\theta \; d\theta$

(b) $\int (x-1)^2 dx$

(c) $\int \frac{2x}{x^2+1} dx$

(d) $\int \frac{1}{\sqrt{4-x^2}} \; dx$

4. (64 points) Use the evaluation theorem as needed to find each of the definite and improper integrals below. Each improper integral must be identified as such to get credit.

   (a) $\displaystyle\int_0^2 \frac{d}{dx}[(x^2-3)(x^3-1)]\,dx$

   (b) $\displaystyle\int_4^9 \frac{9}{\sqrt{x}}dx$

   (c) $\displaystyle\int_0^{\pi/2} \cos x \cos(\sin x)\,dx$

   (d) $\displaystyle\int_3^4 \frac{x-1}{x^2-4}\,dx$

(e) $\displaystyle\int_e^{\infty} (x \ln x)^{-1}\, dx$

(f) $\displaystyle\int_1^4 |x^3 - 6x^2 + 11x - 6|\, dx$. Note that $f(x) = x^3 - 6x^2 + 11x - 6$ factors into $(x-1)(x-2)(x-3)$.

(g) $\displaystyle\int_0^1 x(x-2)^9\, dx$

(h) $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\, dx$

5. (15 points) Construct a triangle with an acute angle θ such that $\tan\theta = x/2$. Then compute each of the following in terms of x.

   (a) $\sin\theta$

   (b) $\sin(2\theta)$

   (c) $\csc\theta$

6. (15 points) For each integral below, use the substitution θ such that $x = 2\tan\theta$ to find an equivalent $d\theta$ integral. Do not evaluate.

   (a) $\displaystyle\int_0^1 \frac{x^2}{\sqrt{4+x^2}}\,dx$

   (b) $\displaystyle\int \frac{1}{4+x^2}\,dx$

   (c) $\displaystyle\int x\sqrt{4+x^2}\,dx$