

October 29, 2004

Name \_\_\_\_\_

On all the following questions, **show your work**. There are 125 points available on this test. Do not try to do all the problems. Try to find four or five that you can do well.

1. (15 points) Find the area enclosed by the curves  $f(x) = x^2 + 1$  and  $g(x) = 16 + x - x^2$ .
2. (15 points) Find the average value of the function  $f(x) = 2x(x^2 - 1)^{3/2}$  over the interval  $[1, \sqrt{5}]$ . Verify that there exists a number  $c$ ,  $1 \leq c \leq \sqrt{5}$  such that  $f(c) = f_{ave}$ . In other words verify that the conclusion of the mean value theorem for integrals is satisfied.
3. (15 points) The elastic force of a spring satisfies Hooke's Law  $F(x) = kx$  with spring stiffness  $k$ . To extend the spring from its equilibrium  $x = 0$  to  $x = 0.20$  meters, a force of 8 Newtons is required. Find  $k$ . Then find the amount of work  $W$  done extended the spring from equilibrium to  $x = 0.80$  meters.
4. (15 points) Use the numerical integration feature on your calculator to find the length of the curve  $y = \sin x$  between 0 and  $\pi$ .

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5. (50 points) In the first quadrant  $x \geq 0, y \geq 0$ , a region  $R$  is bounded above by the curve  $y = 2x - x^2$  and bounded below by  $y = x^3$ .
- (a) Set up an integral whose value is the volume of the solid obtained by rotating  $R$  around the  $x$ -axis.
- (b) Set up an integral whose value is the volume of the solid obtained by rotating  $R$  around the  $y$ -axis.
- (c) Set up an integral whose value is the volume of the solid obtained by rotating  $R$  around the line  $y = -1$ .
- (d) Using the cylindrical shells technique, set up an integral whose value is the volume of the solid obtained by rotating  $R$  around the line  $x = -1$ .
- (e) Using the slicing technique, set up an integral whose value is the volume of the solid obtained by rotating  $R$  around the line  $x = -1$ .
6. (15 points) Calculate the moments  $M_x$  and  $M_y$  and the center of mass of the lamina  $R$  of density  $\rho = 1$  and shape given by  $R = \{(x, y) \mid 0 \leq x \leq 2 \text{ and } 0 \leq y \leq \sqrt{4 - x^2}\}$ .