November 22, 2004   **Name** _____

On all the following questions, **show your work.** There are 148 points available on this test. Do not try to do all the problems. Try to find four or five that you you can do well.

1. (48 points) Convergence of series. If the series converges, name the test you used to verify convergence and demonstrate that it applies. If it fails to converge, state why that is the case.

    (a) $\sum_{n=1}^{\infty} a_n$ where $a_n = 1/n$.

    (b) $\sum_{n=1}^{\infty} a_n$ where $a_n = 1/n^{1.1}$.

    (c) $\sum_{n=0}^{\infty} a_n$ where $a_0 = 1$ and $a_{n+1} = a_n(\sin n/n)$ for $n \geq 0$.

    (d) $\sum_{n=1}^{\infty} a_n$ where $a_n = \ln n/(n^2)$.

    (e) $\sum_{n=0}^{\infty} a_n$ where $a_n = \frac{n+3}{n^3-2}$.

    (f) $\sum_{n=1}^{\infty} a_n$ where $a_n = \cos n/(n^2 + 1)$.

2. (40 points) For each series, find the sum or state that it does not converge.

(a) $\frac{1}{10} + \frac{2}{100} + \frac{1}{1000} + \frac{2}{10000} + \cdots$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4n^2 - 9}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$

(e) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^n$

3. (30 points) Find the interval of convergence for the first and third of the series and the radius of convergence of the second.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n! x^n}{n^n}$.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n (x-3)^n}{(n+1)!}$.

4. (30 points)

   (a) Find the Taylor polynomial T_5 of the function $f(x) = \cos x$ about the point $\pi/2$.

   (b) Find the Taylor series of the function $f(x) = \cos x$ about the point $\pi/2$.

   (c) What is the radius of convergence of the series?

   (d) Use the polynomial T_5 to approximate $\cos(\pi/2 + 0.1)$.

   (e) Estimate the accuracy of the approximation in the part above.