Tuesday, April 26, 2005   Name _____

On all the following questions, **show your work.** There are 155 points available on this test. Do not try to do all the problems. Try to find four or five that you can do well.

1. (5 points) What is the limit of the *sequence* a_n defined by $a_n = \frac{n^2 - 6n + 5}{3n^2 + 2n - 12}$, as $n \to \infty$?

2. (15 points) Consider the series $\sum\limits_{n=1}^{\infty} a_n$ where

$$a_n = \frac{(6n^2 + 4)3^{n+1}}{5^n}$$

Does the series converge absolutely, converge conditionally or diverge. What test did you use? Why is it conclusive?

3. (15 points) Consider the series $\sum\limits_{n=1}^{\infty} a_n$ where

$$a_n = \frac{e^{n-3}}{\sqrt{n+5}(n+2)!}$$

Does the series converge absolutely, converge conditionally or diverge. What test did you use? Why is it conclusive?

4. (20 points) Find the interval of convergence for the given power series.

$$\sum_{n=1}^{\infty} \frac{(x-8)^n}{n(-6)^n}$$

(a) What is the radius of convergence?

(b) Discuss convergence at the left endpoint.

(c) Discuss convergence at the right endpoint.

(d) What is the interval of convergence?

5. (20 points) Consider the power series

$$\sum_{n=1}^{\infty} \frac{(7x)^n}{n^{11}}$$

(a) What is the radius of convergence?

(b) Discuss convergence at the left endpoint.

(c) Discuss convergence at the right endpoint.

(d) What is the interval of convergence?

6. (20 points) Find all the values of x such that the given series would converge.

$$\sum_{n=1}^{\infty} \frac{(x-4)^n}{4^n}$$

(a) What is the radius of convergence?

(b) Discuss convergence at the left endpoint.

(c) Discuss convergence at the right endpoint.

(d) What is the interval of convergence?

7. (20 points) Suppose that the Maclaurin series for $f(x) = \dfrac{5x}{(7+x)}$ is $\sum_{n=0}^{\infty} c_n x^n$.
   Find the coefficients, c_0, c_1, c_2, c_3, and c_4. Find the radius of convergence R of the power series.

8. (10 points) Find the Maclaurin series of the function $f(x) = (9x^2)e^{-x}$. That is, find c_0, c_1, etc. so that $f(x) = \sum_{n=0}^{\infty} c_n x^n$. Hint: first find the series for $f(x) = e^{-x}$

9. (10 points) Find the Maclaurin series of the function $f(x) = (7x^2)\sin(x)$. That is, find c_0, c_1, etc. so that $f(x) = \sum_{n=0}^{\infty} c_n x^n$.

10. (20 points) Suppose $\sum_{n=0}^{\infty} c_n(x-2)^n$ converges at $x = 3$.

    (a) Must it converge for $x = 0$?

    (b) Must it converge for $x = 1$?

    (c) Must it converge for $x = 1.2$?

    (d) What can be said about the radius of convergence R?

    (e) What can be said about the convergence of
    $\sum_{n=0}^{\infty} nc_n(1.5-2)^n$? How is this series related to the one above?

11. (20 points) Find the Taylor polynomial $T_5(x)$ for $f(x) = \sin(2x)$ at $x = \pi$.

b. Find an upper bound for $|R_5(x)|$ on the interval $[\pi/2, 3\pi/2]$.

c. Find the radius of convergence of the Taylor series.