November 27, 2005 **Name** ______________________________

On all the following questions, **show your work.** There are 139 points available on this test. Do not try to do all the problems. Try to find four or five that you you can do well.

1. (10 points) Suppose the series $\sum a_n$ has partial sums S_n given by $S_n = \frac{(2n-1)^2}{(3n+1)^2}$. Does the series converge? If so, to what?

2. (20 points) Test for convergence and find the sum if possible. If you cannot find the sum, state the test you used to determine convergence (or divergence).

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n!}$.

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+(\pi/e)^n}$.

   (d) $\displaystyle\sum_{n=1}^{\infty} \sin(n+1) - \sin(n)$.

   (e) $\displaystyle\sum_{n=1}^{\infty} \arctan(n+1) - \arctan n$.

3. (25 points) Match each of the following with the correct statement.

   A. The series is absolutely convergent.

   C. The series converges, but is not absolutely convergent.

   D. The series diverges.

___1. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{6n+4}$

___2. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(2n)}{n^2}$

___3. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+7}$

___4. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)(6^2-1)^n}{6^{2n}}$

___5. $\displaystyle\sum_{n=1}^{\infty} \frac{(-5)^n}{n^5}$

4. (24 points) The interval of convergence of a power series can be of four forms, $[a,b], (a,b], [a,b)$ and (a,b). For each part below gave an example of a power series with the given interval of convergence.

   (a) $(0,2)$

   (b) $[-1,5]$

   (c) $[1,7)$

5. (20 points) Consider the function $f(x) = e^{2x-1}$.

   (a) Find the Taylor polynomial $T_5(x)$ at $a = 1/2$.

   (b) Find an upper bound for $|R_5(x)|$ on the interval $[0,1]$.

   (c) Find the radius of convergence of the Taylor series.

6. (20 points) Consider the function $f(x) = \frac{1}{1-x^2}$.

   (a) Find a power series representation of $f(x) = \frac{1}{1-x^2}$.

   (b) Differentiate both sides of the equation in (a) to find a power series representation of $f'(x)$ and find the interval of convergence for this series.

7. (20 points) Consider the function $f(x) = 3x^2 \sin(x^2)$. Recall that the Maclaurin series for $\sin x$ is given by $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$.

   (a) Find the Maclaurin series representation of $f(x)$.

   (b) Use part (a) of the problem to find each of the following derivatives of f.

      i. $f^{(3)}(0)$

      ii. $f^{(4)}(0)$

      iii. $f^{(8)}(0)$

      iv. $f^{(12)}(0)$