

April 27, 2006

Name \_\_\_\_\_

On all the following questions, **show your work**. There are 130 points available on this test. Do not try to do all the problems.

1. (20 points) Determine whether each of the **sequences** below converges. If so, find the limit.

(a)  $\lim_{n \rightarrow \infty} \frac{11(2^n)+10}{8(2^n)}$

(b)  $\lim_{n \rightarrow \infty} \frac{35}{3^n} + 2 \arctan(n^2)$

(c)  $\lim_{n \rightarrow \infty} n(0.7)^n$

(d)  $\lim_{n \rightarrow \infty} \frac{(2n+1)^2}{(3n-1)^2}$

2. (25 points) Label each of the following series with C or D, where C stands for Convergent, D stands for Divergent. State the reason for your answer.

(a)  $\sum_{n=1}^{\infty} \frac{n^2}{n^3+3}$

(b)  $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$

(c)  $\sum_{n=1}^{\infty} ne^{-n^2}$

(d)  $\sum_{n=1}^{\infty} \frac{7+5^n}{10^n}$

(e)  $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$

3. (30 points) Test each of the following series for convergence. name the test you use. Two points for right convergence answer, three points for correct test. If a series converges conditionally, but not absolutely, say so.

(a)  $\sum_{n=1}^{\infty} \frac{(2n+2)!}{(n!)^2}$

(b)  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(5+n)4^n}{(n^2)3^{2n}}$

- (c)  $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$
- (d)  $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n\pi}$
- (e)  $\sum_{n=1}^{\infty} (-1)^n n^{-1} \ln(n+3)$
- (f)  $\sum_{n=1}^{\infty} \frac{5 + \sin(n)}{\sqrt{n}}$

4. (10 points) Find the value of

$$\int_2^{\infty} \frac{dx}{(9x-2)^8}$$

Determine whether  $\sum_{n=2}^{\infty} \frac{1}{(9n-2)^8}$  is convergent or divergent.

5. (15 points) Find the interval of convergence for the given power series.

$$\sum_{n=1}^{\infty} \frac{(x-6)^n}{n(-6)^n}$$

In particular, be sure to discuss convergence at the endpoints of the interval.

6. (15 points) Suppose that  $\frac{5x}{(6+x)} = \sum_{n=0}^{\infty} c_n x^n$ .

Find the first few coefficients.

$$c_0 = \underline{\hspace{2cm}}$$

$$c_1 = \underline{\hspace{2cm}}$$

$$c_2 = \underline{\hspace{2cm}}$$

$$c_3 = \underline{\hspace{2cm}}$$

$$c_4 = \underline{\hspace{2cm}}$$

Find the radius of convergence  $R$  of the power series.

$$R = \underline{\hspace{2cm}} .$$

7. (15 points) Consider the series  $1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$  obtained from the harmonic series by replacing every third plus sign with a minus sign. Very few of the theorems in class apply to this series. Its not alternating. Does it converge or diverge? Discuss your reasoning. If you think it converges, does it converge conditionally or absolutely?