The homework includes all except the first problem.

1. In a math contest, three problems, A, B, and C were posed. Among the participants there were 25 who solved at least one problem. Of all the participants who did not solve problem A, the number who solved problem B was twice the number who solved C. The number who solved only problem A was one more than the number who solved A and at least one other problem. Of all participants who solved just one problem, half did not solve problem A. How many solved only problem B?

2. Let $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Let $A = \{1, 2, 3, 4, 5\}, B = \{4, 5, 6, 7\}$, and $C = \{1, 5, 8\}$. Recall that $\times$ denotes Cartesian product and $\overline{X}$ denotes the complement of X with respect to $\mathcal{U}$. Find each of the following. Recall that $A \oplus B = A\backslash B \cup B\backslash A$ denotes the symmetric difference of A and B and that $|X|$ denotes the number of elements of the finite set X.

   (a) $|(\mathcal{U} \times \mathcal{U})\backslash(A \times A)|$

   (b) $|\overline{A} \times \overline{A}|$

   (c) $|(A \times B) \cup (B \times A)|$

   (d) $|\overline{A \cup B \cup C}|$

   (e) $|\,(A \times A) \cap (B \times B) \cap (C \times C)|$

   (f) $|\,(A \times A) \cup (B \times B) \cup (C \times C)|$

   (g) $|\,(A \times B) \cup (B \times C) \cup (C \times A)|$

   (h) $|\,(A \times A) \cup (A \times B) \cup (A \times C)|$

   (i) $|\,(A \times B) \oplus (B \times A)|$

   (j) $|\,(A \oplus B) \times (B \oplus A)|$

   (k) $|\,(A \cup B) \oplus C|$

3. Ask about this one. We might not have time to discuss characteristic functions. Recall that we sometimes write AB for $A \cap B$. Use characteristic functions to show each of the following:

   (a) $(A \oplus B) \oplus C = A \oplus (B \oplus C)$ (associativity of symmetric difference). Hint: First prove that $f_{A \oplus B} = f_A + f_B - 2f_A f_B$.

   (b) $A \cap (\overline{BC}) = (A\overline{B}) \cup (A\overline{C})$

   (c) $A \cup (B \cap \overline{C}) = (A \cup B) \cap (A \cup \overline{C})$.

4. Among 18 students in a room, 7 study mathematics, 10 study science, and 10 study computer programming. Also, 3 study mathematics and science, 4 study mathematics and computer programming, and 5 study science and computer programming. We know that 1 student studies all three subjects. How many of these students study none of the three subjects?

5. Let A, B, and C be sets with the following properties:

   - $|A| = 100$, $|B| = 50$, and $|C| = 48$
   - The number of elements that belong to exactly one of the three sets is twice the number that belong to exactly two of the sets.

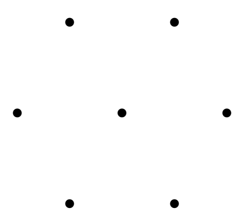
- The number of elements that belong to exactly one of the three sets is three times the number that belong to all of the sets.

How many elements belong to all three sets?

6. Three sets A, B, and C have the following properties: $N(A) = 63, N(B) = 91, N(C) = 44, N(A \bigcap B) = 25, N(A \bigcap C) = 13, N(C \bigcap B) = 11$. Also, $N(A \bigcup B \bigcup C) = 153$. What is $N(A \bigcap B \bigcap C)$?

7. Two distinct circles and a triangle are given in a plane. What is the largest number of points that can belong to at least two of the three figures? By a circle we mean just the circumference, by a triangle we mean just the union of the edges.

8. How many equilateral triangles in the same plane as the lattice shown have at least two vertices in the hexagonal lattice?



9. How many integers in the set $\{1, 2, 3, 4, \ldots, 360\}$ have at least one prime divisor in common with 360?

10. Let $U = \{1, 2, 3, \ldots, 1000\}$ and let A_2, A_3, and A_5 denote the subsets of U defined as follows:
$A_2 = \{n \mid 1 \leq n \leq 1000 \text{ and } n \text{ is even }\}$,
$A_3 = \{n \mid 1 \leq n \leq 1000 \text{ and } n \text{ is a multiple of } 3\}$,
$A_5 = \{n \mid 1 \leq n \leq 1000 \text{ and } n \text{ is a multiple of } 5\}$,
All complements are taken with respect to U. Find the number of elements of each of the sets listed below. a. $A_2 \bigcap A_3 \bigcap A_5$; b. $A_2 \bigcap A_3 \bigcap \overline{A_5}$; c. $A_2 \bigcap \overline{A_3} \bigcap A_5$; d. $\overline{A_2} \bigcap A_3 \bigcap A_5$; e. $A_2 \bigcap \overline{A_3} \bigcap \overline{A_5}$; f. $\overline{A_2} \bigcap A_3 \bigcap \overline{A_5}$; g. $\overline{A_2} \bigcap \overline{A_3} \bigcap A_5$; and h. $\overline{A_2} \bigcap \overline{A_3} \bigcap \overline{A_5}$.

11. In a survey of the chewing gum tastes of a group of baseball players, it was found that: 22 liked juicy fruit
    25 liked spearmint
    39 like bubble gum
    9 like both spearmint and juicy fruit
    17 liked juicy fruit and bubble gum
    20 liked spearmint and bubble gum
    6 liked all three
    4 liked none of these
    How many baseball players were surveyed?

12. Mr. Brown raises chickens. Each can be described as thin or fat, brown or red, hen or rooster. Four are thin brown hens, 17 are hens, 14 are thin chickens, 4 are thin hens, 11 are thin brown chickens, 5 are brown hens, 3 are fat red roosters, 17 are thin or brown chickens. How many chickens does Mr. Brown have?

13. Consider the following information regarding three sets A, B, and C all of which are subsets of a set U. If $N(S)$ denotes the number of members of S, suppose that $N(A) = 14$, $N(B) = 10$, $N(A \cup B \cup C) = 24$ and $N(A \cap B) = 6$. Consider the following assertions:

    1. C has at most 24 members
    2. C has at least 6 members
    3. $A \cup B$ has exactly 18 members

    Which ones are true?