1. For each of the following problems, let $S(n) = n$ in case n is a single digit integer. If $n \geq 10$ is an integer, $S(n)$ is the sum of the digits of n. Similarly $P(n)$ is n if n is a positive single digit integer and the product of the digits of n otherwise. If there is not solution, prove it.
   (a) What is the smallest solution to $S(n) = 2005$. Express your answer in exponential notation.
   (b) How many five-digit numbers n satisfy $S(S(n)) + S(n) = 50$.
   (c) Find all solutions to $S(S(S(n))) + S(S(n)) + S(n) = 100$.
   (d) Find all solutions to $S(S(n)) + S(n) + n = 2007$.
   (e) (2007 North Carolina High School Math Contest) Find the sum $S(1) + S(2) + \cdots + S(2007)$.
2. Recall the two functions *floor* ($\lfloor\rfloor$) and *fractional part* ($\langle\rangle$), defined by $\lfloor x \rfloor$ is the largest integer that is less than or equal to x, and $\langle x \rangle = x - \lfloor x \rfloor$.
   (a) For each member x of the set S, $S = \{\pi, 1.234, -1.234, \frac{7}{3}, -\frac{7}{3}\}$, evaluate $\langle x \rangle$ and $\lfloor x \rfloor$.
   (b) Define another function f by $f(x) = x - 10\lfloor \frac{x}{10} \rfloor$. Find $f(x)$ and $f(\lfloor x \rfloor)$ for each x in S.
   (c) Let $g(x) = \lfloor \frac{\lfloor x \rfloor}{10} \rfloor - 10\lfloor \frac{\lfloor x \rfloor}{100} \rfloor$. Evaluate g at each of the members of S.
   (d) Prove that for any number $x = 100a + 10b + c + f$, where a is a positive integer, b is a digit, c is a digit, and $0 \leq f < 1$, $g(x) = b$. In other words, $g(x)$ is the tens digit of x.
3. Let $a, b, c, d,$ and e be digits satisfying $4 \cdot \underline{abcde4} = \underline{4abcde}$. Find all five of the digits.
4. (a) Grab a calculator. (You might not be able to do this one in your head.)
   (b) Key in the first three digits of your phone number (NOT the area code)
   (c) Multiply by 80.
   (d) Add 1.
   (e) Multiply by 250.
   (f) Add the last 4 digits of your phone number.
   (g) Add the last 4 digits of your phone number AGAIN.
   (h) Subtract 250.

(i) Divide number by 2.

Explain why you get such an interesting answer.

5. This problem is part of an email message I received from a friend. "This is pretty neat. Can you figure out the trick? I usually just accept these kinds of things and enjoy them rather than trying to second-guess them:
OUT TO DINNER MATHEMATICS
This is pretty neat how it works out. DON'T CHEAT BY SCROLLING DOWN FIRST It takes less than a minute....... Work this out as you read. Don't cheat and read the bottom until you've worked through it! This is fun!

   (a) First of all, pick the number of times a week that you would like to have dinner out. (try for more than once but less than 10)
   (b) Multiply this number by 2 (Just to be bold)
   (c) Add 5. (Just because)
   (d) Multiply it by 50 - I'll wait while you get the calculator
   (e) If you have already had your birthday this year add 1755.... If you haven't, add 1754.
   (f) Now subtract the four digit year that you were born.

   You should have a three digit number. The first digit of this was your original number (How many times you want to eat out each week.) The next two numbers are YOUR AGE! (Oh YES, it IS!!!!!) THIS IS THE ONLY YEAR (2005) IT WILL EVER WORK, SO SPREAD IT AROUND WHILE IT LASTS. IMPRESSIVE, ISN'T IT?" The problem is to explain why this works.

6. A two-digit integer N that is not a multiple of 10 is k times the sum of its digits. The number formed by interchanging the digits is m times the sum of the digits. What is the relationship between m and k?

7. The fido challenge: http://digicc.com/fido/

8. The crystal cabobble challenge: http://www.sinotrading.us/crystalball.htm

9. How many two-digit positive integers N have the property that the sum of N and the number obtained by reversing the order of the digits of N is a perfect square?

10. A check is written for x dollars and y cents, both x and y two-digit numbers. In error it is cashed for y dollars and x cents, the incorrect amount exceeding the correct amount by \$17.82. Find a possible value for x and y.

11. Solve the alpha-numeric problem $\underline{2abc} \times 4 = \underline{cba2}$, where a, b and c are decimal digits.

    In problems 12 and 13, we are dealing with six-digit numbers.

12. The rightmost digit of a six-digit number N is moved to the left end. The new number obtained is five times N. What is N?

13. Repeat the same problem with the 5 changed to a 4. That is $4(\underline{abcdef}) = \underline{fabcde}$

14. Let $N = 1234567891011\ldots19992000$ be the integer obtained by appending the decimal representations of the numbers from 1 to 2000 together in order. What is the remainder when N is divided by 9?