Throughout we use both the notations $\binom{n}{r}$ and C_r^n for the number $\frac{n!}{(n-r)!r!}$.

1. A *falling* number is an integer whose decimal representation has the property that each digit except the units digit is larger than the one to its right. For example 96521 is a falling number but 89642 is not. How many n-digit falling numbers are there, for $n = 1, 2, 3, 4, 5, 6, 7, 8,$ and 9? What is the total number of falling numbers of all sizes?

2. Cyprian writes down the middle number in each of the $\binom{9}{5} = 126$ five-element subsets of $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Then he adds all these numbers together. What sum does he get?

3. Counting sums of subset members.

   (a) How many numbers can be expressed as a sum of two or more distinct members of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$?

   (b) How many integers can be expressed as a sum of two or more different members of the set $\{0, 1, 2, 4, 8, 16, 32\}$?

   (c) How many numbers can be expressed as a sum of four distinct members of the set $\{17, 21, 25, 29, 33, 37, 41\}$?

   (d) How many numbers can be expressed as a sum of two or more distinct members of the set $\{17, 21, 25, 29, 33, 37, 41\}$?

   (e) How many integers can be expressed as a sum of two or more distinct elements of the set $\{1, -3, 9, -27, 81, -243\}$?

4. How many of the first 242 positive integers are expressible as a sum of three or fewer members of the set $\{3^0, 3^1, 3^2, 3^3, 3^4\}$ if we are allowed to use the same power more than once. For example, $5 = 3 + 1 + 1$ can be represented, but 8 cannot. Hint: think about the ternary representations.

5. John has 2 pennies, 3 nickels, 2 dimes, 3 quarters, and 8 dollars. For how many different amounts can John make an exact purchase (with no change required)?

6. How many positive integers less than 1000 have an odd number of positive integer divisors?

7. An urn contains marbles of four colors: red, white, blue, and green. When four marbles are drawn without replacement, the following events are equally likely:

(a) the selection of four red marbles;

(b) the selection of one white and three red marbles;

(c) the selection of one white, one blue, and two red marbles; and

(d) the selection of one marble of each color.

What is the smallest number of marbles that the urn could contain?

8. How many squares in the plane have two or more vertices in the set $S = (0,0), (0,1), (0,2), (1,0), (1,1), (2,0)$.

9. Numbers with a given digit sum.

   (a) How many numbers in the set $\{100, 101, 102, \ldots, 999\}$ have a sum of digits equal to 9?

   (b) How many four digit numbers have a sum of digits 9?

   (c) How many integers less than one million have a sum of digits equal to 9?