Class Problems, Second week, Math 6105

1. Consider the game of Bouton's nim with pile sizes $15, 20, 25, 30, 35$.

   (a) Find the binary representation of each pile size.

   (b) Find the binary configuration of the game. That is, write these binary numbers in a column and compute their nim sum; Remember that to compute the nim sum, add the numbers with the understanding that $1 + 1 = 0, \quad 0 + 0 = 0, \quad 1 + 0 = 0 + 1 = 1$, and there is no carry from one column to another.

   (c) Notice that the binary configuration is not balanced since the nim sum of the pile sizes is not zero. Find a move which results in a balanced binary configuration. Is there just one such move or are there several?

   (d) Suppose you made a move which balances the configuration. Assume your opponent takes one counter from the same pile as the one from which you removed counters. What move do you make now?

2. Answer the same questions about each of the Bouton's Nim games listed below.

   (a) $N(16, 17, 18)$

   (b) $N(19, 27, 38)$

   (c) $N(16, 17, 18, 19, 27, 38)$

3. Consider the games $N_i(k)$ and $N_d(k)$ for the various values of k. Recall that $N_i(k)$ and $N_d(k)$ refer to dynamic one pile nim, where i is the identity function and d is the doubling function. The *positions* in the game are ordered pairs (t, k) where t is the pile size and k is the maximum number of counters that can be removed on the next turn. A *move* is an ordered pair of positions $(t, k) \mapsto (t - r, f(r))$, where $r \leq k$. The position $(t - r, f(r))$ results from taking r counters from the pile of t counters. The moves in $N_i(k)$ are ordered pairs $(t, k) \mapsto (t - r, r)$ where $r \leq k$ and the moves in $N_d(k)$ are ordered pairs $(t, k) \mapsto (t - r, 2r)$ where $r \leq k$. For each of the positions below, determine whether the position is safe or unsafe. If it is unsafe, find a move that results in a safe position.

   (a) In $N_i(k)$ the position $100, 99$.

   (b) In $N_i(k)$ the position $200, 199$.

   (c) In $N_i(k)$ the position $400, 199$.

(d) In $N_i(k)$ the position $320, 50$.

(e) In $N_d(k)$ the position $100, 99$.

(f) In $N_d(k)$ the position $200, 199$.

(g) In $N_d(k)$ the position $400, 199$.

(h) In $N_d(k)$ the position $320, 50$.