Practice Final Exam, Math 6105

1. Use the repeated subtraction method to find the base 4 representation of each of the following numbers

   (a) 83

   (b) 13.125

2. Use the method of repeated multiplication to find a base 4 representation of each of the following numbers

   (a) 0.375

   (b) 129/256

3. Find the base -4 representation of each of the following numbers

   (a) 193

   (b) 117.125

4. Find the Fibonacci representation of each of the following numbers

   (a) 193

   (b) 280

5. You're playing the game $N_d(k)$ and your opponent has just left you the position $(200, 6)$. Do you have a good move? Explain.

6. You're playing the game $N_i(k)$ and your opponent has just left you the position $(200, 6)$. Do you have a good move? Explain.

7. Consider the game of Bouton's nim with pile sizes $19, 24, 25, 26, 31$.

   (a) Find the binary representation of each pile size.

   (b) Find the binary configuration of the game. That is, write these binary numbers in a column and compute their nim sum.

   (c) Notice that the binary configuration is not balanced since the nim sum of the pile sizes is not zero. Find a move which results in a balanced binary configuration. Is there just one such move or are there several?

   (d) Suppose you made a move which balances the configuration. Assume your opponent takes one counter from the same pile as the one from which you removed counters. What move do you make now?

8. Find the number of positive integer divisors of the number 13!. Explain how you got your answer.

9. Find the remainder when each of the following numbers is divided by 18.

   (a) 123,456,789,101,112

   (b) 5^{2004}

   (c) $3^{2001} \cdot 5^{2004} \cdot 7^{2005}$

10. Find all the divisors of the number $N = 2^5 3^4 5$. How many even divisors does N have? How many of N's divisors are multiples of 6?

11. Let $\mathcal{U} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the universal set. Let $S = \{1, 2, 3, 4, 5\}$ and $T = \{4, 5, 6, 7, 8\}$.

    (a) How many four-element subsets A of $\mathcal{U}$ satisfy $|A \cap S| = 2$ and $|A \cap T| = 2$?

    (b) Let D denote the set of all four-digit numbers that can be built using the elements of S as digits and allowing repetition of digits. What is $|D|$?

(c) How many elements of D have four different digits?

(d) How many elements of D have exactly three different digits?

(e) How many even numbers belong to D?

12. Prove that geometrical progression is increasing faster than perfect squares. Specifically: prove that for appropriate $n_0 > 0$ and any $n \geq n_0$

$$2^n > n^2.$$

13. Solve the decanting problem for containers of sizes 139 and 149; that is find integers x and y satisfying $139x + 149y = d$ where d is the GCD of 139 and 149.

14. Find a relation R on the set $S = \{1, 2, 3\}$ satisfying each of the following conditions. Find one relation for each part.

    (a) R_1 has exactly 3 ordered pairs members and is transitive.

    (b) R_2 has exactly 3 ordered pairs members and is not transitive.

    (c) R_3 is symmetric and has exactly 5 ordered pairs members.

    (d) R_4 is an equivalence relation with exactly 5 ordered pairs members.

    (e) R_5 is a partially ordered set with exactly 4 ordered pairs members.