

UNIVERSITY OF NORTH CAROLINA CHARLOTTE
1998 HIGH SCHOOL MATHEMATICS CONTEST
March 9, 1998

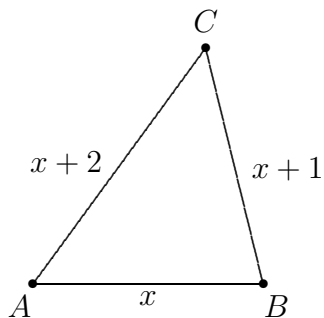
1. Find the sum

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{10 \cdot 11}.$$

- (A) 1 (B) 2 (C) $\frac{10}{11}$ (D) $\frac{11}{12}$ (E) none of A, B, C or D
2. The mean test score in a math class with 27 students was 72. A student who scored 85 was moved to another class. What was the mean score of the remaining 26 students?
- (A) 69.5 (B) 70 (C) 70.5 (D) 71 (E) 71.5
3. Suppose the value of a new car declines linearly over a ten year period from the original value of \$20,000 to the value \$2,000. What is the value of the car after six years?
- (A) \$8,800 (B) \$9,200 (C) \$11,000
(D) \$12,800 (E) \$13,200
4. Which of the five fractions has the smallest value?
- (A) $\frac{250,386,765,412}{250,384,765,412}$ (B) $\frac{250,384,765,412}{250,383,765,412}$ (C) $\frac{250,385,765,412}{250,384,765,412}$
(D) $\frac{250,386,765,412}{250,385,765,412}$ (E) $\frac{250,387,765,412}{250,386,765,412}$
5. The square of an integer is called a *perfect square*. If n is a positive perfect square, which of the following represents the largest perfect square less than n ?
- (A) $n - 1$ (B) n (C) $n^2 - 2n + 1$ (D) $n^2 + n$ (E) $n - 2\sqrt{n} + 1$
6. Let $f(x) = x^3 + kx - 3$. For what value of k is $x - 1$ a factor of $f(x)$?
- (A) $-1/2$ (B) $1/2$ (C) 1 (D) $3/2$ (E) 2

7. In triangle ABC , $AB = x$, $BC = x + 1$, and $AC = x + 2$. Which of the following **must** be true?

- i. $x \geq 1$
- ii. $x \leq 5\sqrt{2}$
- iii. $\angle C \leq 60^\circ$



- (A) i only (B) ii only (C) iii only
(D) i and ii only (E) i and iii only

8. Each of the cards shown below has a number on one side and a letter on the other. How many of the cards must be turned over to prove the correctness of the statement:

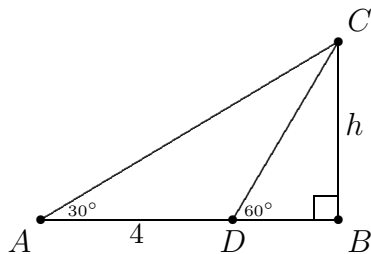
Every card with a vowel on one side has a prime number on the other side.



- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

9. In right triangle ABC , the point D on \overline{AB} is 4 units from A , $\angle CDB = 60^\circ$ and $\angle CAB = 30^\circ$. What is the altitude h ?

(A) 3 (B) $2\sqrt{3}$ (C) $\sqrt{14}$ (D) 4 (E) $3\sqrt{2}$



10. The numbers x, y , and z satisfy

$$|x + 2| + |y + 3| + |z - 5| = 1.$$

Which of the following could be $|x + y + z|$?

(A) 0 (B) 2 (C) 5 (D) 7 (E) 10

11. Suppose $a < 0$ and $|a| \cdot x \leq a$. Evaluate $|x + 1| - |x - 2|$.

(A) -3 (B) -1 (C) $-2x + 1$ (D) $2x + 3$ (E) 3

12. The set of points satisfying the three inequalities

$$y \geq 0, \quad y \leq x, \quad \text{and} \quad y \leq 6 - x/2$$

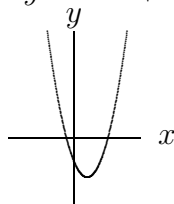
is a triangular region with an area of

(A) 12 (B) 18 (C) 24 (D) 36 (E) 48

13. If $x > 5$, which of the following is smallest?

(A) $\frac{5}{x}$ (B) $\frac{5}{x-1}$ (C) $\frac{x}{5}$ (D) $\frac{5}{x+1}$ (E) $\frac{x+1}{5}$

14. The graph of the quadratic function $y = ax^2 + bx + c$ is shown below.



Which of the following is true?

- (A) $ac < 0$ and $ab < 0$ (B) $ac < 0$ and $ab > 0$
(C) $ac > 0$ and $ab < 0$ (D) $ac > 0$ and $ab > 0$
(E) At least one of a, b , and c could be zero.
15. A drawer contains exactly six socks—two are green, two are red, two are blue. If two socks are selected at random without replacement, what is the probability that they match?
- (A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$
16. Of the members of three athletic teams at Harding High, 21 are on the basketball team, 26 are on the baseball team, and 29 are on the football team. A total of 14 play both baseball and basketball; 15 play both baseball and football; and 12 play football and basketball. There are eight who play all three sports. How many students play on at least one of the teams?
- (A) 43 (B) 49 (C) 51 (D) 76 (E) 84
17. Five test scores have a mean (average score) of 91, a median (middle score) of 92 and a mode (most frequent score) of 95. The sum of the two lowest test scores is
- (A) 172 (B) 173 (C) 174 (D) 178 (E) 179

18. In his last will, a farmer asked that his horses be distributed among his four sons. The oldest was to get one third of the herd, the second oldest, one fourth of the herd, and each of the two youngest ones was to get one fifth of the herd. When the sons read the will, they were puzzled because none of them were going to get an integer number of horses. At that moment, they discovered that a baby horse had just been born. Each son would receive an integer number of horses, but the baby horse would be left over. How many horses did the farmer have originally?

- (A) 29 (B) 59 (C) 89 (D) 119 (E) 239

19. If a, b, c and d are four positive numbers such that $\frac{a}{b} < \frac{c}{d}$, then

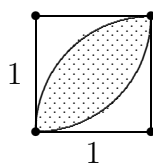
(A) $ab < dc$ (B) $a + c < b + d$ (C) $a + d < b + c$

(D) $\frac{a+c}{b+d} < \frac{c}{d}$ (E) $\frac{c-a}{d-b} < \frac{c}{d}$

20. How many odd three-digit numbers have three digits different?

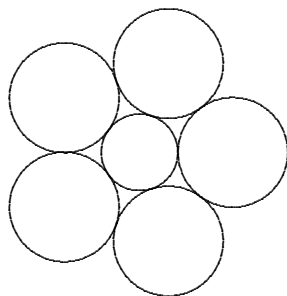
- (A) 60 (B) 288 (C) 300 (D) 320 (E) 405

21. What is the area of the region common to two unit circles whose centers are $\sqrt{2}$ apart?



- (A) $\frac{1}{2} \left(1 - \frac{\pi}{4}\right)$ (B) $\frac{\pi}{2}$ (C) $1 - \frac{\pi}{4}$ (D) $\frac{\pi}{2} - 1$ (E) $\frac{1}{2}$

22. Five circles with equal radii are situated in the plane so that each is tangent to two others and externally tangent to a unit circle. Find the radius of each of the five circles, rounded to two decimal places.
- (A) 1.39 (B) 1.41 (C) 1.43 (D) 1.45 (E) 1.47



23. If Sam and Peter are among 6 men who are seated at random in a row, the probability that exactly 2 men are seated between them is
- (A) $1/10$ (B) $1/8$ (C) $1/5$ (D) $1/4$ (E) $4/15$
24. Four congruent triangular corners are cut off an 11×13 rectangle. The resulting octagon has eight edges of equal length. What is this length?
- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7
25. Evaluate the continued fraction

$$2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}$$

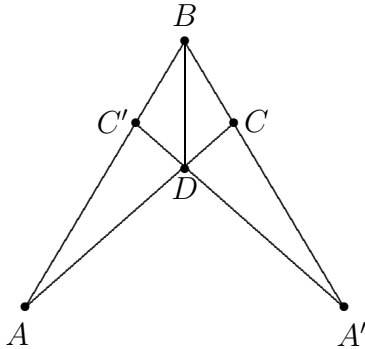
- (A) $\sqrt{5}$ (B) $\sqrt{6}$ (C) $1 + \sqrt{2}$ (D) $1 + \sqrt{3}$ (E) 3
26. Let x and b be positive integers. Suppose that x is represented as 324 in base b , and x is represented as 155 in base $b + 2$. What is b ?
- (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

27. It is known that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. What is the value of

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}?$$

- (A) $\frac{\pi^2}{36}$ (B) $\frac{\pi^2}{12}$ (C) $\frac{\pi^2}{8}$ (D) $\frac{\pi^2}{7}$ (E) $\frac{2\pi^2}{9}$

28. In triangle ABC , $AB = 20$, $BC = 5$, and $\angle ABC = 60^\circ$. The triangle is reflected in the plane about the bisector of angle ABC to produce a new triangle $A'BC'$ as shown. What is the area of the region enclosed by the union of the two triangles?



- (A) $20\sqrt{3}$ (B) $24\sqrt{3}$ (C) $40\sqrt{2}$ (D) $40\sqrt{3}$ (E) $50\sqrt{2}$