# UNC Charlotte 2007 Algebra

March 5, 2007

1. We randomly select 4 prime numbers without replacement from the first 10 prime numbers. What is the probability that the sum of the four selected numbers is odd?

   **(A)** 0.21 **(B)** 0.30 **(C)** 0.36 **(D)** 0.40 **(E)** 0.50

2. What is the area (in square units) of the region in the first quadrant defined by $18 \leq x + y \leq 20$?

   **(A)** 36 **(B)** 38 **(C)** 40 **(D)** 42 **(E)** 44

3. How many four-digit numbers between 6000 and 7000 are there for which the thousands digits equal the sum of the other three digits?

   **(A)** 20 **(B)** 22 **(C)** 24 **(D)** 26 **(E)** 28

4. How many positive two-digit integers have an odd number of positive divisors?

   **(A)** 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

5. If $x$ is positive, what is the least value of $x + \frac{9}{x}$?

   **(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 6

6. The area of an annular region bounded by two concentric circles is $5\pi$ square centimeters. The difference between the radii of the circles is one centimeter. What is the radius of the smaller circle, in centimeters?

   **(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 6

7. If we divide 344 by $d$ the remainder is 3, and if we divide 715 by $d$ the remainder is 2. Which of the following is true about $d$?

   **(A)** $10 \leq d \leq 19$ **(B)** $20 \leq d \leq 29$ **(C)** $30 \leq d \leq 39$

   **(D)** $40 \leq d \leq 49$ **(E)** $50 \leq d \leq 59$

8. The sum $a + b$, the product $a \cdot b$ and the difference of squares $a^2 - b^2$ of two positive numbers $a$ and $b$ is the same nonzero number. What is $b$?

   **(A)** 1 **(B)** $\frac{1+\sqrt{5}}{2}$ **(C)** $\sqrt{3}$ **(D)** $\frac{7-\sqrt{5}}{2}$ **(E)** 8

9. It takes Amy and Bill 15 hours to paint a house, it takes Bill and Chandra 20 hours, and it takes Chandra and Amy 30 hours. How long will it take if all three work together?

   **(A)** 9 hours and 40 minutes **(B)** 10 hours **(C)** 12 hours

   **(D)** 13 hours and 20 min **(E)** 14 hours

10. Maya deposited 1000 dollars at 6% interest compounded annually. What is the number of dollars in the account after four years?

    **(A)** \$1258.47 **(B)** \$1260.18 **(C)** \$1262.48

    **(D)** \$1263.76 **(E)** \$1264.87

11. A peddler is taking eggs to the market to sell. The eggs are in a cart that holds up to 500 eggs. If the eggs are removed from the cart either $2, 3, 4, 5$, or 6 at a time, one egg is always left over. If the eggs are removed 7 at a time, no eggs are left over. Let $n$ denote the number of eggs in the cart. Which of the following is true about $n$?

    **(A)** $n \in [1, 100]$ **(B)** $n \in [101, 200]$ **(C)** $n \in [201, 300]$

    **(D)** $n \in [301, 400]$ **(E)** $n \in [401, 500]$

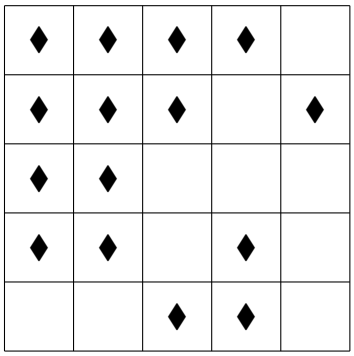
12. An athlete covers three consecutive miles by swimming the first, running the second and cycling the third. He runs twice as fast as he swims and cycles one and a half times as fast as he runs. He takes ten minutes longer than he would if he cycled the whole three miles. How many minutes does he take?

    **(A)** 16 **(B)** 22 **(C)** 30 **(D)** 46 **(E)** 70

13. How many 5-digit numbers can be built using the digits $1, 2$, and 3 if each digit must be used at least once?

    **(A)** 60 **(B)** 90 **(C)** 120 **(D)** 150 **(E)** 243

14. What is the fewest crickets that must hop to new locations so that each row and each column has three crickets? Crickets can jump from any square to any other square.



**(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

15. How many real number solutions does the equation $5\sqrt{x} = 6 - x$ have?

**(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** None of these

16. A quadrilateral $ABCD$ has vertices with coordinates $A\,(0,0)$, $B\,(6,0)$, $C\,(5,4)$, $D\,(3,6)$. What is its area?

**(A)** 18 **(B)** 19 **(C)** 20 **(D)** 21 **(E)** 22

17. Let $A$ be the area of a triangle with sides $5, 5$, and $8$, and let $B$ denote the area of a triangle with sides $5, 5$, and $6$. Which of the following statements is true?

**(A)** $A < B < 12$ **(B)** $B < A < 12$ **(C)** $A = B$

**(D)** $12 < A < B$ **(E)** $12 < B < A$

18. In a course "Leadership in Mathematics" there are several tests. Each test is worth 100 points. After the last test John realized that if he had received 97 points for the last test, his average score for the course would have been a 90, and that if he had made a 73, his average score would have been an 87. How many tests are there in the course?

**(A)** 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

19. There are four consecutive integers such that the sum of the cubes of the first three numbers equals the cube of the fourth number. Find the sum of the four numbers.

    **(A)** 12 **(B)** 16 **(C)** 18 **(D)** 22 **(E)** 24

20. Which of following statements is true about the equation $|x^2-2x-3| = x+2$?

    **(A)** There are no solutions. **(B)** There is only one solution.

    **(C)** There are exactly two solutions. **(D)** There are exactly three solutions.

    **(E)** There are exactly four solutions.

21. Let $a, b, c \geq 2$ be natural numbers and

$$a^{(b^c)} = (a^b)^c?$$

    Which one(s) of $a, b, c$ can have arbitrary values.

    **(A)** $a$ **(B)** $b$ **(C)** $c$ **(D)** both $a$ and $b$ **(E)** all three

22. Suppose $\log_8 a + \log_8 b = \log_8 a \cdot \log_8 b$ and $\log_a b = 3$. What is the value of $a$?

    **(A)** 9 **(B)** $b^3$ **(C)** 16

    **(D)** 32 **(E)** There is not enough information to answer

23. A movie was so awful that one-half of the audience left after a few minutes. Five minutes later, one-third of the remaining audience left. Ten minutes later, one-fourth of those remaining left, leaving only nine people in the audience. Let $N$ denote the number of people in the audience at the beginning of the movie? Then

    **(A)** $N \in [1, 20]$ **(B)** $N \in [21, 30]$ **(C)** $N \in [31, 40]$

    **(D)** $N \in [41, 50]$ **(E)** $N > 50$

24. The four angles of a quadrilateral are in arithmetic progression and the largest is twice the smallest. What is the largest angle?

    **(A)** $60^\circ$ **(B)** $90^\circ$ **(C)** $100^\circ$ **(D)** $120^\circ$ **(E)** $140^\circ$

25. The sum of three numbers is 155. The same number $k$ is obtained using any of the three operations below:

    a. Seven is added to the smallest number.

    b. Seven is subtracted from the middle number.

    c. The largest number is divided by 3.

    How much greater is the largest than the smallest?

    **(A)** 57 **(B)** 63 **(C)** 65 **(D)** 67 **(E)** 69

26. A standard deck of 52 cards has four *suits*, clubs, diamonds, hearts, and spades. Each suit has 13 *values* including ace, two, three, ..., ten, and three face cards, Jack, Queen, and King. What is the fewest number of cards that must be selected from a deck to guarantee that the set contains three-of-a-kind; that is, three cards of the same value?

    **(A)** 15 **(B)** 18 **(C)** 20 **(D)** 25 **(E)** 27

27. Let $N = \underline{abcde}$ denote the five digit number with digits $a, b, c, d, e$ and $a \neq 0$. Let $N' = \underline{edcba}$ denote the reverse of $N$. Suppose that $N > N'$ and that $N - N' = 5x014$ where $x$ is a digit. What is $x$?

    **(A)** 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

28. A sequence of three real numbers forms an arithmetic progression with a first term of 9. If 2 is added to the second term and 20 is added to the third term, the three resulting numbers form a geometric progression. What is the smallest possible value for the third term of the geometric progression?

    **(A)** 1 **(B)** 4 **(C)** 36 **(D)** 49 **(E)** 81

29. Betty has 6 daughters and no sons. Some of her daughters have 6 daughters, and the rest have none. Betty has a total of 30 daughters and granddaughters, and no great-granddaughters. How many of Betty's daughters and granddaughters have no daughters?

    **(A)** 24 **(B)** 25 **(C)** 26 **(D)** 27 **(E)** 28

30. How many digits are required to represent the number $2007^{2007}$ in decimal form?

    **(A)** 6628 **(B)** 6629 **(C)** 6630 **(D)** 6631 **(E)** 7321