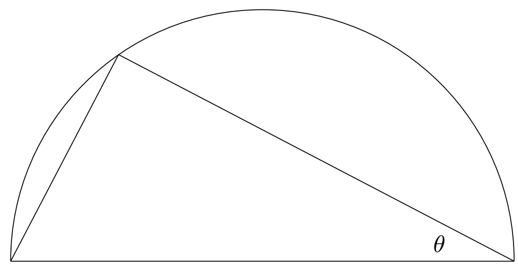
March 2, 2015

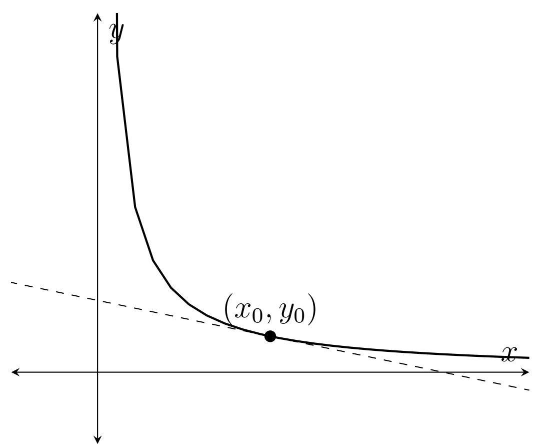
1. A triangle is inscribed in a semi-circle of radius r as shown in the figure:



The area of the triangle is

(A) $r^2 \sin 2\theta$ (B) $\pi r^2 - \sin\theta$ (C) $r \sin\theta \cos\theta$ (D) $\pi r^2/4$ (E) $\pi r^2/2$

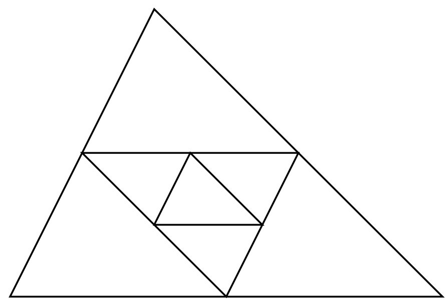
2. A triangle is formed by the coordinate axes and the tangent line at the point with coordinates (x_0, y_0) lying on the curve $\mathscr{C} = \{(x, y) : xy = k\}$, as shown in the figure.



The numbers x_0, y_0, and the slope m of the tangent line satisfy $y_0 = -mx_0$. The area of the triangle is

(A) $2k$ (B) $2k^2$ (C) $x_0 + 2k$ (D) $k(x_0 + y_0)$ (E) $k/(x_0 + y_0)$

3. There are three triangles of different sizes: small, medium and large. The small one is inscribed in the medium one such that its vertices are at the midpoints of the three edges of the medium one. The medium triangle is inscribed in the large triangle in the same way as shown in the figure. If the small triangle has area 1, what is the sum of the areas of the three triangles?



   (A) 14 (B) 16 (C) 19 (D) 21 (E) 25

4. Describe the shape of the graph of $f(x) = \dfrac{6x^2 + 7x - 3}{2x + 3}$.

   (A) hyperbola (B) parabola (C) circle (D) full line (E) line with a hole

5. How many of elements of the set $\{-12.3, -9, -5, 1, 2.14, 3.6, 5.2, 7.8, 101\}$ are **not** solutions of the equation $|2x^2 - 9x + 6| = 2x^2 - 9x + 6$?

   (A) 2 (B) 3 (C) 5 (D) 6 (E) 8

6. The number of elements in the intersection of the solution set of $\dfrac{\sqrt{2} \cdot x - 1}{2x^2 - 4\sqrt{2} \cdot x + 4} > 0$ with the set $\{-7, -5, -2, -1, 1, \sqrt{2}, \sqrt{107}, \sqrt{108}\}$ is

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

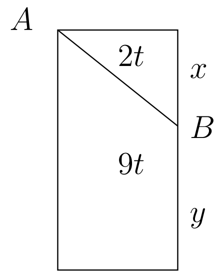
7. How many positive integers are in the range of the function $f(x) = -2x^2 + 8x - 3$?

   (A) 0 (B) 2 (C) 3 (D) 5 (E) 6

8. The number of elements in the intersection of $A = \left\{-1, -\dfrac{1}{2}, 0, \dfrac{1}{2}, 1, \dfrac{3}{2}, 2\right\}$ with the solution set of $\left(1 + \dfrac{x+2}{x-1}\right)\left(1 + \dfrac{x-3}{2}\right) > 0$ is

   (A) 01 (B) 1 (C) 2 (D) 3 (E) 4

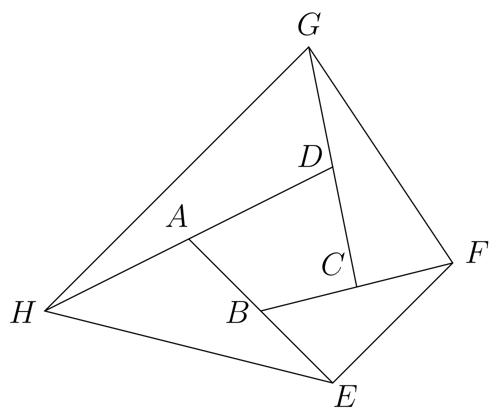
9. The line segment AB divided the rectangle in the picture into two parts. The proportion of the areas of these parts is $2:9$.



In what proportion are the lengths x and y?

(A) $2:9$ (B) $2:7$ (C) $4:7$ (D) $4:9$ (E) $5:9$

10. Consider a convex 4-gon $ABCD$. Reflect A about B to get E, B about C to get F, C about D to get G and D about A to get H. What percentage is the area of $ABCD$ of the resulting 4-gon $EFGH$?



(A) 10% (B) 20% (C) 25% (D) 30% (E) 33.5%

11. What is the number of real solutions of the equation $2015x^6+4030x^3+x^2+2x+2016=0$ if we count each different solution only once, **without multiplicities**?

(A) 0 (B) 1 (C) 2 (D) 4 (E) 6

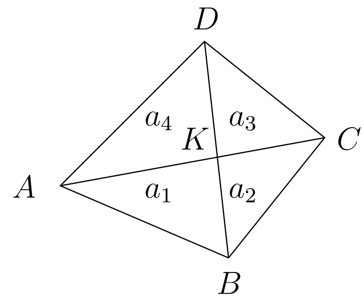
12. How many four-digit positive integers are there with exactly two even and two odd, pairwise different digits?

(A) 2008 (B) 2100 (C) 2160 (D) 2240 (E) 2640

13. What is the nth digit after the decimal point in the decimal representation of $(\sqrt{26}-5)^n$.

(A) 0 (B) 1 (C) 6 (D) 5 (E) Can not be determined

14. The diagonals of the convex four-gon $ABCD$ divide the four-gon into four triangles, whose areas are a_1, a_2, a_3, and a_4 respectively, as shown in the figure.



Which of the following identities must hold for these areas?

(A) $a_1 + a_2 = a_3 + a_4$ (B) $a_1 + a_3 = a_2 + a_4$ (C) $a_1 \cdot a_2 = a_3 \cdot a_4$ (D) $a_1 \cdot a_3 = a_2 \cdot a_4$ (E) $a_1 - a_3 = a_2 - a_4$

15. The lengths of 3 edges, meeting in one vertex, of a rectangular box, measured in centimeters are distinct positive integers. One of the edges has length 6 cm. What is the sum of the lengths of the other two edges if the number expressing the volume of the box in cubic centimeters is the same as the number expressing its surface area in square centimeters.

(A) 10 (B) 12 (C) 14 (D) 16 (E) 20

16. The National Assembly of a country has 10 committees. Every member of the National Assembly works in exactly two committees and any pair of committees has exactly one member in common. How many members does the National Assembly have?

(A) 25 (B) 30 (C) 45 (D) 50 (E) 60

17. What is the sum of all integers n, for which $Q(n) = \dfrac{4n^2 - 4n - 24}{n^3 - 3n^2 - 4n + 12}$ is also an integer?

(A) 10 (B) 11 (C) 12 (D) 13 (E) infinity

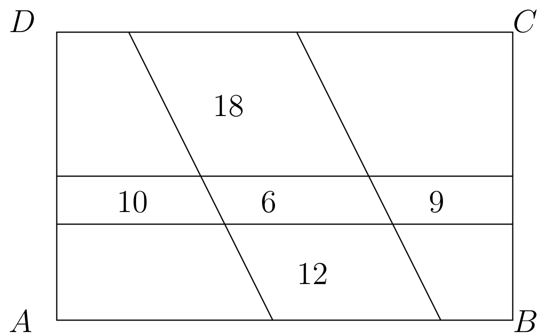
18. We draw all diagonals of a convex 12-gon. Each intersection of diagonals is contained in exactly two diagonals. How many intersections of diagonals are there inside the 12-gon?

(A) 216 (B) 480 (C) 495 (D) 500 (E) 512

19. What is the **exact** value of $\dfrac{\sin(3\alpha) + \sin\alpha}{\sin(2\alpha)\cos(\alpha)}$ (for all values of α such that the above expression is defined)?

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

20. We subdivide the rectangle $ABCD$ with two pairs of parallel lines into 9 parts, as shown in the figure. The areas of five of these parts, measured in square centimeters is provided. What is the area of the rectangle $ABCD$, measured in square centimeters?



(A) 150 (B) 190 (C) 180 (D) 155 (E) 200

21. What is the sum of the squares of the solutions of the equation $\log_3(x) + \log_x(9) = 3$?

(A) 0 (B) 9 (C) 12 (D) 81 (E) 90

22. In the language BadSpeak, the alphabet contains only the letters a, b, and c. How many 4-letter words in BadSpeak contain the letter "c"?

(A) 81 (B) 108 (C) 108 (D) 65 (E) 1

23. If $\log x = -8$ and $\log y = 14$, then $\log x^2y^3 =$?

(A) -112 (B) 26 (C) 12544 (D) 6 (E) 0

24. Let w be a real number. What is the sum of the (possibly complex) roots of the equation $x^2 + 13x + w = 0$?

(A) w (B) $-w$ (C) $13 + w$ (D) $-13 - w$ (E) -13

25. An $a \times b \times c$ block of unit cubes has faces with surface areas $48, 60$, and 80. Suppose the six faces are painted, after which the block is cut into unit cubes. Which of the following is the probability that a randomly selected cube is painted on exactly two faces?

(A) 1/2 (B) 3/20 (C) 2/21 (D) 7/60 (E) 1/8