March 7, 2016

1. The little tycoon Johnny says to his fellow capitalist Annie, "If I add 7 dollars to 3/5 of my funds, I'll have as much capital as you have." To which Annie replies, "So you have only 3 dollars more than me." How much money does Annie have?

   (A) 9 (B) 17 (C) 22 (D) 15 (E) 20

2. Four little girls, Katrina, Helen, Marie, and Naomi, sang in a concert. Each song was sung by three girls. Katrina sang 8 songs, which was more than the other girls. Helen sang 5 songs, which was fewer than the other girls. How many songs were sung in the concert?

   (A) 27 (B) 9 (C) 10 (D) 6 (E) 15

3. A state program allows people to collect empty milk bottles and exchange them for bottles full of milk. Four empty bottles may be exchanged for one full bottle. How many bottles of milk can a family drink having initially 24 empty bottles?

   (A) 6 (B) 7 (C) 8 (D) 9 (E) 12

4. Nick left Nicktown at 10:18 a.m. and arrived at Georgetown at 1:30 p.m., walking at a constant speed. On the same day, George left Georgetown at 9:00 a.m. and arrived at Nicktown at 11:40 a.m., walking at a constant speed along the same road. The road crosses a wide river. Nick and George arrived at the bridge simultaneously, each from his side of the river. Nick left the bridge one minute later than George. When did they arrive at the bridge?

   (A) 10 : 45 (B) 11 : 20 (C) 11 : 00 (D) 2 : 10 (E) 1 : 30

5. A man is filling two tanks with water using two hoses. When the smaller tank is half full, he switches hoses. He keeps filling the tanks, and they both fill up completely at the same moment. The smaller tank is filled first from the less powerful hose. The first hose delivers water at the rate of 2.9 liters per minute, the second at a rate of 8.7 liters per minute. What is the volume of the larger tank if the volume of the smaller tank is 12.6 liters?

   (A) 19.3 (B) 18.2 (C) 25.2 (D) 30 (E) 21

6. Let a and b be the two solutions to the equation

$$2x^2 - 3x - 3 = 0.$$

Find the value $\frac{1}{a} + \frac{1}{b}$.

(A) 2 (B) 1 (C) 0 (D) -1 (E) 5

7. What is the sum of the digits of all numbers from 1 to 1000?

   (A) 13501 (B) 13601 (C) 13701 (D) 13801 (E) 13901

8. What is the area of the region of the plane determined by the inequality $7 \leq |x| + |y| \leq 13$?

   (A) 169 (B) 81 (C) 240 (D) 120 (E) 78

9. A square pyramid is built from unit cubes with an $n \times n$ base and each successive square is one less on the side. So, for example the second layer is $(n-1) \times (n-1)$. What is the smallest n for which the volume is more than $72 \cdot 10^6$?

   (A) 550 (B) 600 (C) 620 (D) 621 (E) 625

10. A chemist has a solution consisting of 5 ounces of propanol and 17 ounces of water. She would like to change the solution into a 60% propanol solution by adding z ounces of propanol. Which of the following equations should she solve in order to determine the value of z?

    (A) $5/(z+17) = .6$ (B) $(z+5)/22 = .6$ (C) $(z+5)/17 = .6$ (D) $(z+5)/(z+17) = .6$ (E) $(z+5)/(z+22) = .6$

11. How many integers n satisfy $|n^3 - 222| < 666$?

    (A) 11 (B) 15 (C) 17 (D) 19 (E) 20

12. There is an undeveloped chain of 20 small islands in the Pacific Ocean. What is the largest number of bridges that could be built among these islands such that there is at most one way to get from any island to any other island?

    (A) 10 (B) 12 (C) 19 (D) 20 (E) 190

13. Seven women and five men attend a party. At this party each man shakes hands with each other person once. Each woman shakes hands only with men. How many handshakes took place at the party?

    (A) 31 (B) 35 (C) 40 (D) 42 (E) 45

14. Which of the five fractions is smallest?

    (A) $\frac{250388749}{250388751}$ (B) $\frac{250388748}{250388750}$ (C) $\frac{250388747}{250388749}$ (D) $\frac{250388750}{250388751}$ (E) $\frac{250388751}{250388752}$

15. The midpoints of the sides of a triangle are $(2,4),(4,5)$ and $(3,2)$. One vertex of this triangle is :

    (A) $(5,5)$ (B) $(4,7)$ (C) $(3,7)$ (D) $(3,5)$ (E) $(5,7)$

16. A total of n cards numbered 1 through n is divided into two stacks. What is the minimum value of n such that at least one stack will include a pair of cards whose numbers add up to an exact square?

    (A) 36 (B) 15 (C) 52 (D) 48 (E) 12

17. A bug crawls along the edges of a cube. Each time it gets to a vertex, it chooses one of the three edges leaving that vertex. How many of the $3^4 = 81$ paths of length 4 lead back to the original vertex?

    (A) 81 (B) 4 (C) 8 (D) 21 (E) 37

18. Let $a_0 = 10$, and for each $n > 0$, let $a_n = 100a_{n-1}+(n+10)$. For how many n, $0 \leq n \leq 100$ is it true that a_n is a multiple of 3?

    (A) 62 (B) 65 (C) 67 (D) 71 (E) 77

19. For a regular tetrahedron $ABCD$, a plane P is called a middle plane if all four distances from the vertices A, B, C, and D to the plane P are the same. How many middle planes are there for a given tetrahedron?

    (A) 1 (B) 3 (C) 4 (D) 6 (E) 7

20. The outside of an $a \times b \times c$ block of unit cubes is painted, where $a < b < c$. Exactly two-thirds of the abc cubes have some paint. Which of the following could be (a, b, c)?
    (A) (5,7,9) (B) (6,8,10) (C) (7,9,11) (D) (6,10,12) (E) (7,10,12)

21. What is the remainder when the product $N = 1008 \cdot 1009 \cdot 1010$ is divided by 77?

    (A) 18 (B) 27 (C) 42 (D) 63 (E) 65

22. A fair die is rolled 6 times. Let p denote the probability that each of the six faces on the die appears exactly once among the six rolls. Which of the following is correct?

    (A) $p \leq 0.02$ (B) $0.02 < p \leq 0.04$ (C) $0.04 < p \leq 0.06$ (D) $0.06 < p \leq 0.08$
    (E) $0.1 < p$

23. Find the value of the expression

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots + \frac{2015}{2016!}$$

(A) $\dfrac{2015!}{2016!}$ (B) $1-\dfrac{2015!}{2016!}$ (C) $\dfrac{2015}{2016}$ (D) $1-\dfrac{1}{2016}$ (E) $1-\dfrac{1}{2016!}$

24. Let x and y be two positive real numbers satisfying

$x+y+xy=10$ and $x^2+y^2=40$. What integer is nearest to the positive value of $x+y$?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

25. If a, b, c, and d are nonzero real numbers, $a/b = c/d$, and $a/d = b/c$, then which one of the following must be true?

(A) $a=\pm b$ (B) $a=\pm c$ (C) $a=\pm d$ (D) $b=\pm c$ (E) none of A, B, C or D

26. A 3 by 3 by 3 wooden cube is painted on all 6 faces and then cut into 27 unit cubes. One unit cube is randomly selected and rolled. What is the probability that exactly two of the five visible faces are painted?

(A) $\frac{1}{27}$ (B) $\frac{2}{27}$ (C) $\frac{12}{27}$ (D) $\frac{15}{27}$ (E) $\frac{5}{81}$