

# UNC Charlotte 2008 Algebra

March 3, 2008

- The sum of all divisors of 2008 is  
(A) 8    (B) 1771    (C) 1772    (D) 3765    (E) 3780
- From the list of all natural numbers  $2, 3, \dots, 999$ , delete nine sublists as follows. First, delete all even numbers except 2, then all multiples of 3 except 3, then all multiples of 5 except 5, and so on, for the nine primes  $2, 3, 5, 7, 11, 13, 17, 19, 23$ . Find the sum of the **composite** numbers left in the remaining list.  
(A) 0    (B) 899    (C) 961    (D) 2701    (E) 3062
- The polynomial  $P(x) = (x^6 - 1)(x - 1) - (x^3 - 1)(x^2 - 1)$  has potentially 7 real zeros. Which of the following is a zero of multiplicity greater than 1?  
(A)  $-2$     (B)  $-1$     (C)  $0$     (D)  $1$     (E)  $2$
- New York City and Washington D.C. are about 240 miles apart. A car leaves New York City at noon traveling directly south toward Washington D.C. at 55 miles per hour. At the same time and along the same route, a second car leaves Washington D.C. bound for New York City traveling directly north at 45 miles per hour. How far has the car which left New York City traveled when the drivers meet for lunch at 2:24 P.M.?  
(A) 128 miles    (B) 130 miles    (C) 131 miles    (D) 132 miles    (E) 134 miles
- Suppose  $x + 1/y = 1.5$  and  $y + 1/x = 3$ . What is  $x \div y$ ?  
(A) 0.2    (B) 0.3    (C) 0.4    (D) 0.5    (E) 1.2
- If  $x + y + z = 7$  and  $x^2 + y^2 + z^2 = 21$ , what is  $xy + yz + zx$ .  
(A) 10    (B) 11    (C) 12    (D) 13    (E) 14

7. During a rebuilding project by contractors A, B, and C, there was a shortage of tractors. The contractors lent each other tractors as needed. At first, A lent B and C as many tractors as they each already had. A few months later, B lent A and C as many as they each already had. Still later, C lent A and B as many as they each already had. By then each contractor had 24 tractors. How many tractors did contractor A originally have?
- (A) 21    (B) 24    (C) 30    (D) 33    (E) 39
8. In a school of 20 teachers, 10 teach Humanities, 8 teach Social Studies and 6 teach Sciences; 2 teach Humanities and Social Studies, but none teach Social Studies and Sciences. How many teach Humanities and Sciences?
- (A) 2    (B) 3    (C) 4    (D) 5    (E) 6
9. A certain integer  $N$  has exactly eight factors, counting itself and 1. The numbers 35 and 77 are two of the factors. What is the sum of the digits of  $N$ ?
- (A) 9    (B) 10    (C) 16    (D) 18    (E) 20
10. Suppose that  $x$  and  $y$  are positive real numbers that satisfy the equations  $x^2 + xy + y^2 = 7$  and  $3x + y = 3$ . Find  $y - 4x$ .
- (A) 1    (B) 2    (C) 3    (D)  $3/2$     (E)  $5/2$
11. Suppose  $a, b,$  and  $c$  are integers satisfying
- $$\begin{aligned}a + b^2 + 2ac &= 22 \\b + c^2 + 2ab &= 36 \\c + a^2 + 2bc &= -2\end{aligned}$$
- What is  $a + b + c$ ?
- (A)  $-6$     (B)  $-2$     (C) 4    (D) 7    (E) 9
12. Two points  $A$  and  $B$  are 4 units apart are given in the plane. How many lines in the plane containing  $A$  and  $B$  are 2 units from  $A$  and 3 units from  $B$ ?
- (A) 0    (B) 1    (C) 2    (D) 3    (E) 4

13. The number of real solutions of the equation  $|x - 2| + |x - 3| = 3$  is  
(A) 1    (B) 2    (C) 3    (D) 4    (E) *many*
14. The numbers  $1 \leq a, b, c, d, e \leq 2008$  are randomly chosen integers (repetition is allowed). What is the probability that  $abc + de$  is even?  
(A)  $1/2$     (B)  $1/4$     (C)  $11/16$     (D)  $7/16$     (E)  $21/32$
15. On a fence are sparrows and pigeons. When five sparrows leave, there remain two pigeons for every sparrow. After that twenty-five pigeons leave, and the ratio of sparrows to pigeons becomes three to one. Find the original number of birds.  
(A) 44    (B) 48    (C) 50    (D) 54    (E) 60
16. Suppose  $a$  and  $b$  are digits satisfying  $1 < a < b < 8$ . Also, the sum  $1111 + 111a + 111b + \dots$  of the smallest eight four-digit numbers that use only the digits  $\{1, a, b, 8\}$  is 8994. What is  $a + b$ ?  
(A) 6    (B) 7    (C) 8    (D) 9    (E) 10
17. At one of mayor Pat McCrory's parties, each man shook hands with everyone except his spouse, and no handshakes took place between women. If 13 married couples attended, how many handshakes were there among these 26 people?  
(A) 78    (B) 185    (C) 234    (D) 312    (E) 325
18. On a die, 1 and 6, 2 and 5, 3 and 4 appear on opposite faces. When 2 dice are thrown, multiply the numbers appearing on the top and bottom faces of the dice as follows:  
(a) number on top face of 1st die  $\times$  number on top face of 2nd die  
(b) number on top face of 1st die  $\times$  number on bottom face of 2nd die  
(c) number on bottom face of 1st die  $\times$  number on top face of 2nd die  
(d) number on bottom face of 1st die number  $\times$  on bottom face of 2nd die.

What can be said about the sum  $S$  of these 4 products?

- (A) The value of  $S$  depends on luck and its expected value is 48
- (B) The value of  $S$  depends on luck and its expected value is 49
- (C) The value of  $S$  depends on luck and its expected value is 50
- (D) The value of  $S$  is 49
- (E) The value of  $S$  is 50

19. During recess, one of five pupils wrote something nasty on the chalkboard. When questioned by the class teacher, the following ensued:

A : It was 'B' or 'C'.

B : Neither 'E' nor I did it.

C : You are both lying.

D : No, either A or B is telling the truth.

E : No, 'D', that is not true.

The class teacher knows that three of them never lie while the other two cannot be trusted. Who was the culprit?

- (A) A    (B) B    (C) C    (D) D    (E) E

20. How many distinct real number solutions does  $(3x^2 + 2x)^2 = (x^2 + 2x + 1)^2$  have?

- (A) 0    (B) 1    (C) 2    (D) 3    (E) 4

21. Let  $N$  be the largest 7-digit number that can be constructed using each of the digits 1, 2, 3, 4, 5, 6, and 7 such that the sum of each two consecutive digits is a prime number. What is the remainder when  $N$  is divided by 7?

- (A) 0    (B) 1    (C) 2    (D) 3    (E) 4

22. For how many  $n$  in  $\{1, 2, 3, \dots, 100\}$  is the tens digit of  $n^2$  odd?

- (A) 16    (B) 17    (C) 18    (D) 19    (E) 20

23. How many pairs of positive integers  $(a, b)$  with  $a + b \leq 100$  satisfy

$$\frac{a + b^{-1}}{a^{-1} + b} = 13?$$

- (A) 2    (B) 3    (C) 4    (D) 5    (E) 7

24. The numbers 1, 2, 4, 8, 16, 32 are arranged in a multiplication table, with three along the top and the other three down the column. The multiplication table is completed and the sum of the nine entries is tabulated. What is the largest possible sum obtainable.

- (A) 902    (B) 940    (C) 950    (D) 980    (E) 986

$\times$	$a$	$b$	$c$
$d$			
$e$			
$f$			