Combinatorics Problems

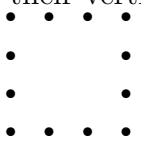
1. How many circles in the plane contain at least three of the points $(0, 0,), (0, 1)$, $(0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2)$?

2. Nine points are distributed around a circle in such a way that when all $\binom{9}{2}$ pairs of points are connected by segments, no three of the segments are concurrent. How many points of intersection are there inside the circle?

3. Two points are randomly and simultaneously selected from the 4×5 grid of 20 lattice points $\{(m, n) | 1 \leq m \leq 5 \; and \; 1 \leq n \leq 4\}$. What is the probability that the distance between them is rational?

4. A *falling* number is an integer whose decimal representation has the property that each digit except the units digit is larger than the one to its right. For example 96521 is a falling number but 89642 is not. How many n-digit falling numbers are there, for $n = 1, 2, 3, 4, 5, 6,$ and 7?

5. Twelve lattice points are arranged along the edges of a 3×3 square as shown. How many triangles have all three of their vertices among these points?



6. How many numbers can be expressed as a sum of four distinct members of the set $\{17, 21, 25, 29, 33, 37, 41\}$?

7. How many numbers can be obtained as the product of two or more of the numbers $3, 4, 4, 5, 5, 6, 7, 7, 7$?

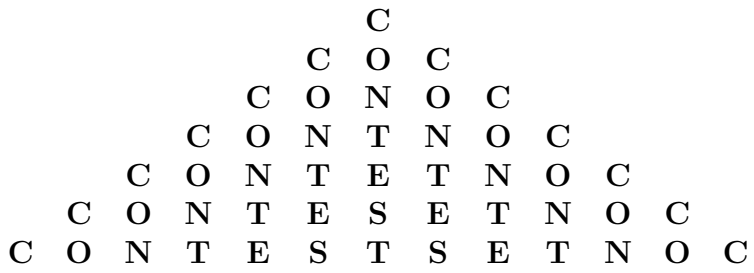
8. (1994 UNC Charlotte Comprehensive Exam) How many of the first 100 positive integers are expressible as a sum of three or fewer members of the set $\{3^0, 3^1, 3^2, 3^3, 3^4\}$ if we are allowed to use the same power more than once. For example, 5 can be represented, but 8 cannot.

9. How many integers can be expressed as a sum of two or more different members of the set $\{0, 1, 2, 4, 8, 16, 31\}$?

10. John has 2 pennies, 3 nickels, 2 dimes, 3 quarters, and 8 dollars. For how many different amounts can John make an exact purchase?

11. What is the size of the largest subset, S, of $\{1, 2, 3, \ldots, 50\}$ such that no pair of distinct elements of S has a sum divisible by 7?

12. How many paths consisting of a sequence of horizontal and/or vertical line segments with each segment connecting a pair of adjacent letters in the diagram below, is the word **CONTEST** spelled out as the path is traversed from beginning to end?



13. Recall that a Yahtzee Roll is a roll of five indistinguishable dice.
    a. How many different Yahtzee Rolls are possible?
    b. How many Yahtzee Rolls have exactly 3 different numbers showing?

14. How many six digit numbers
    a. consist of six different digits?
    b. consist of five different digits?
    c. consist of three odd and three even digits?
    d. have six different digits with no two even digits adjacent?
    e. have four distinct odd digits and two distinct even digits which are not adjacent?

15. How many four digit numbers $\underline{abcd}$ satisfy $|a - d| = 2$?

16. A box contains 2 pennies, 4 nickels, and 6 dimes. Six coins are drawn without replacement, with each of the 12 coins having the same probability of being chosen. What is the probability that the value of the coins is at least 50 cents?

17. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
    a. How many five element subsets does the set have?
    b. How many subsets of S have an odd number of members?
    c. How many subsets of S have 1 as a member?
    d. How many subsets have 1 as a member and do not have 2 as a member?

18. Imagine that the 4×7 grid below represents the streets of a part of the city where you live. You must walk 11 blocks to get from the lower left corner at A to the upper right corner at B.

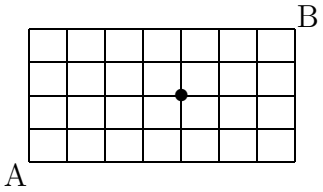
    (a) How many different 11 block walks are there?

    (b) How many 11 block walks avoid the terrible corner marked with the bullet?

    (c) How many 11 block walks go through the terrible corner?

    (d) How many different 12 block walks are there from A to B?

    (e) How many different 13 block walks are there from A to B?



19. How many positive integers less than 1000 have an odd number of positive integer divisors?

20. How many integers can be obtained as a sum of two or more of the numbers $1, 3, 5, 10, 20, 50, 82$?

21. Sixteen lattice points are arranged in a 3×3 square as shown. How many triangles have all three of their vertices among these points?