

UNC Charlotte 2005 Algebra

March 7, 2005

1. What is the y -coordinate of the vertex of the parabola $y = x^2 - 6x + 4$?
(A) -5 (B) -3 (C) 3 (D) 5 (E) 9

2. The line $y = x$ intersects the circle $(x - 3)^2 + (y - 2)^2 = 1$ in two points. What is the sum of the x -coordinates of the two points?
(A) 3 (B) 4 (C) $9/2$ (D) 5 (E) $11/2$

3. The mean of the 7-member list $x, 3x - 4, 4x - 3, -16, 9, 5x + 2$, and $x - 2$ is 4. What is the median of the list?
(A) 5 (B) 6 (C) 6.5 (D) 8 (E) 9

4. Suppose we have two thermometers, one which measures temperature F in degrees Fahrenheit and one which measures temperature C in degrees Celsius. The readings are related by the formula $F = \frac{9}{5}C + 32$. Is it possible to be at a location where both thermometers give the same numerical reading? If so, which of the locations below is the most likely place for this to happen?
- (A) There is no such location
- (B) at the North Pole (very cold)
- (C) at the equator (very hot)
- (D) in North Carolina (moderate temperature)
- (E) all of these locations are equally likely
5. During one quarter of the school year, Kristen took 3 tests. On the second test her grade increased by 25% compared to the first test and then on the third test her grade decreased by 23% compared to the second test. Then with respect to the first test, her grade on the third test
- (A) increased by about 2% (B) increased by about 1%
- (C) decreased by about 2% (D) decreased by about 4%
- (E) stayed almost the same
6. Find the largest possible integer n such that $1 + 2 + 3 + \cdots + n \leq 200$.
- (A) 14 (B) 17 (C) 19 (D) 21 (E) 23
7. The polynomial $p(x) = 2x^4 - x^3 - 7x^2 + ax + b$ is divisible by $x^2 - 2x - 3$ for certain values of a and b . What is the sum of a and b ?
- (A) -34 (B) -30 (C) -26 (D) -18 (E) 30
8. The numbers x and y satisfy $2^x = 15$ and $15^y = 32$. What is the value of xy ?
- (A) 3 (B) 4 (C) 5 (D) 6 (E) none of A, B, C or D

9. One hundred monkeys have 100 apples to divide. Each adult gets three apples while three children share one. How many adult monkeys are there?

(A) 10 (B) 20 (C) 25 (D) 30 (E) 33

10. Let x and y be positive integers satisfying

$$\frac{1}{x+1} + \frac{1}{y-1} = 5/6.$$

Find $x + y$.

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

11. Let x and y be two integers that satisfy all of the following properties:

- (a) $5 < x < y$,
- (b) x is a power of a prime and y is a power of a prime, and
- (c) the quantities $xy + 3$ and $xy - 3$ are both primes.

Among all the solutions, let (x, y) be the one with the smallest product. Which of the following statements is true?

(A) $x + y$ is a perfect square (B) the number xy is prime

(C) $y = x + 3$ (D) $y = x + 1$ (E) $x + y = 17$

12. The product of three consecutive positive integers is eight times their sum. What is the sum of their squares?

(A) 50 (B) 77 (C) 110 (D) 149 (E) 194

13. If $ab = 999$ and both a and b are two digit positive integers, what is $a + b$?

(A) 64 (B) 66 (C) 120 (D) 336 (E) 1000

14. Let $P(x)$ be a polynomial with $P(1) = 1$ and $P(x) = P(x - 1) + x^3$ for all real x . Calculate $P(-3)$.

(A) -36 (B) -9 (C) 1 (D) 9 (E) 36

15. Solve the equation $\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}} = 3$ for x .
- (A) $4/3$ (B) $5/3$ (C) $7/5$ (D) $9/5$ (E) none of A, B, C or D
16. Which of the following is closest to the smallest positive rational number that is an integer multiple of the numbers $\frac{10}{21}$, $\frac{5}{14}$, and $\frac{6}{7}$?
- (A) 1.43 (B) 2.79 (C) 3.43 (D) 4.29 (E) 5.74
17. Suppose all three of the points $(-2, 10)$, $(1, -8)$, and $(4, 10)$ lie on the graph of $y = ax^2 + bx + c$. What is abc ?
- (A) -24 (B) 0 (C) 12 (D) 24 (E) 48
18. An amount of \$2000 is invested at $r\%$ interest compounded continuously. After four years, the account has grown to \$2800. Assuming that it continues to grow at this rate for 16 more years, how much will be in the account?
- (A) \$8976.47 (B) \$9874.23 (C) \$10001.99
- (D) \$10756.48 (E) \$2004.35
19. A cubic polynomial $p(x)$ with leading coefficient 1 has three zeros, $x = 1$, $x = -1$, and $x = 3$. What is the value $p(2)$?
- (A) -3 (B) -1 (C) 1 (D) 2 (E) 3
20. Which of the following numbers is closest to the sum of the roots of the given equation?

$$|2x - 5| = x^2 - 2x - 2$$

- (A) -1.25 (B) -0.75 (C) 0.35 (D) 1.15 (E) 2.12

21. Let p denote the smallest prime number greater than 200 for which there are positive integers a and b satisfying

$$a^2 + b^2 = p.$$

What is $a + b$?

- (A) 16 (B) 17 (C) 18 (D) 19 (E) 20

22. Two rational numbers r and s are given. The numbers $r + s$, $r - s$, rs , and s/r are computed and arranged in order by value to get the list $1/3, 3/4, 4/3, 7/3$. What is the sum of the squares of r and s ?
- (A) $9/25$ (B) $4/9$ (C) $9/4$ (D) $25/9$ (E) 6
23. Suppose x, y, z , and w are real numbers satisfying $x/y = 4/7$, $y/z = 14/3$, and $z/w = 3/11$. When $(x + y + z)/w$ is written in the form m/n where m and n are positive integers with no common divisors bigger than 1, what is $m + n$?
- (A) 20 (B) 26 (C) 32 (D) 36 (E) 37
24. An integer is randomly selected from the set $\{100, 101, \dots, 999\}$. What is the probability that the sum of its digits is the same as the product of its digits?
- (A) 0 (B) $1/900$ (C) $1/300$ (D) $1/150$ (E) $1/100$
25. Four positive integers a, b, c and d satisfy $abcd = 10!$. What is the smallest possible sum $a + b + c + d$?
- (A) 170 (B) 175 (C) 178 (D) 183 (E) 185
26. What is the units digit of the (decimal representations of the) product of all the prime numbers less than 500?
- (A) 0 (B) 1 (C) 2 (D) 5 (E) 6
27. In a 10-team baseball league, each team plays each of the other teams 18 times. No game ends in a tie, and, at the end of the season, each team is the same positive number of games ahead of the next best team. What is the greatest number of games that the last place team could have won?
- (A) 27 (B) 36 (C) 54 (D) 72 (E) 90

List of Primes between 1 and 500:

2	3	5	7	11	13	17	19	23	29
31	37	41	43	47	53	59	61	67	71
73	79	83	89	97	101	103	107	109	113
127	131	137	139	149	151	157	163	167	173
179	181	191	193	197	199	211	223	227	229
233	239	241	251	257	263	269	271	277	281
283	293	307	311	313	317	331	337	347	349
353	359	367	373	379	383	389	397	401	409
419	421	431	433	439	443	449	457	461	463
467	479	487	491	499					