UNC Charlotte 2013 Algebra Exam

March 4, 2013

1. Mystery gifts are offered for sale in boxes, bags, and bundles. If 3 boxes and 2 bundles cost \$46, while 4 bags and 2 boxes cost \$44, and 1 box and 3 bags cost \$28, what is the total cost of 4 boxes, 5 bags and 3 bundles?

(A) \$91 (B) \$94 (C) \$97 (D) \$99 (E) \$99.20

2. The distance a falling object travels in the first t seconds is $16t^2$ feet. In exhibition diving, one person dives from a platform 20 feet above the water and a second person dives from a platform 10 feet above the water. How much later must the person on the lower platform dive after the person on the upper platform does so that they hit the water at the same time?

(A) $\left(\sqrt{20}-\sqrt{10}\right)/4$ (B) $\sqrt{5}/4$ (C) $\sqrt{8}-\sqrt{5}$ (D) $5/8$ (E) $\sqrt{20}-\sqrt{10}$

3. Let $T = \left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{5}\right)\cdots\left(1+\frac{1}{k}\right)$. For how many choices of k in the range $3 \le k \le 100$ is T an integer?

(A) 0 (B) 22 (C) 32 (D) 33 (E) more than 33

4. Sally is trying to determine the number of seats in her schools auditorium. She notices that the first row contains 10 seats, the second row contains 12 seats, the third row contains 14 seats, and the fourth row contains 16 seats. The pattern continues for each row – the next one up always has 2 more seats. If there are 21 rows in the auditorium, how many seats does the auditorium contain?

(A) 610 (B) 620 (C) 630 (D) 640 (E) 650

5. The number $N = 4ab3 + 3b95$ is a multiple of 99. What is the product of the two digits a and b?

(A) 0 (B) 6 (C) 12 (D) 16 (E) 24

6. Arthur takes a walk every day. On the first day of each month, he walks one mile. Each day afterward in the same month, he walks half the total distance he covered so far in the previous days of that month. On what day (after his walk that day) will Arthur's total distance exceed 10 miles for the month?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

7. If each root of the quadratic equation $x^2 + px + q = 0$ is a positive real number and p is at least -6, then q is at most which of the following values?

(A) 3 (B) 5 (C) 7 (D) 9 (E) 10

8. In an interstellar store, a customer is buying construction materials for his granddaughter who is going to build a 6-dimensional cube. The customer only needs the edges for the cube. A big sign says edges are on sale for 2 ISD (interstellar dollar) each. The grandfather (using a cheat sheet provided by the granddaughter) asks for the correct number of edges, but is surprised at the total price. The clerk explains that even though he is buying only edges, he is required by law to also pay for the vertices necessary to build the cube. If each vertex costs 1 ISD, what did the grandfather pay?

(A) 288 ISD (B) 448 ISD (C) 576 ISD (D) 768 ISD (E) 832 ISD

9. Let $T = \{0, 1, 2, 3, 5, 7, 11\}$. How many different numbers can be obtained as the product of three different members of T?

(A) 10 (B) 12 (C) 15 (D) 20 (E) 21

10. How many points in the plane satisfy both $x^2y - y^3 = 0$ and $y = x^2 - 3$?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

11. The diagonals of the sides of a rectangular box have lengths $\sqrt{13}$, $\sqrt{10}$ and $\sqrt{5}$. What is the volume?

(A) 5 (B) $\sqrt{30}$ (C) 6 (D) $\sqrt{50}$ (E) $\sqrt{65}$

12. Mr Green sells apples for \$1.50 each at the local Farmers Market and Ms Blue sells slightly smaller apples for \$1 each. One day Ms Blue had to leave early so she asked Mr Green to manage her stall as the two were side-by-side. To make calculations easier, Mr Green mixed the apples together and changed the signs to read "5 apples for \$6". At that point they had the same number of apples left. By the end of the day he had sold all the apples, but oddly (to him) when he compared how much each would have made by selling separately and how much he had in the till, he found he was 80 dollars short. He had no clue what the problem was, so he split the money evenly and apologized to Ms Blue for messing things up. Certainly, at least one of them lost money. Did both lose money on the deal, or did one come out ahead, and how much did each lose/gain?

(A) Blue lost \$120, Green made \$40 extra (B) Blue lost \$32 and Green lost \$48
(C) Both lost \$40 (D) Blue made \$80 extra and Green lost \$160
(E) Blue made \$160 extra, Green lost \$240

13. Consider the following equation where m and n are real numbers:

$$(x^2 - 2x + m)(x^2 - 2x + n) = 0.$$

Suppose the four roots of the equation form an arithmetic sequence with the first (and smallest) term being $1/4$. What is the value of $|m - n|$?

(A) $3/8$ (B) $1/2$ (C) $5/8$ (D) $3/4$ (E) 1

14. What is the units digit of 3^{2013}?

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

15. Front tires of a car wear out after $45,000$ miles, while the rear tires wear out after $75,000$ miles. After how many miles should you rotate the tires (front↔back) to drive the maximal distance for the set?

(A) $27,125$ (B) $28,125$ (C) $29,125$ (D) $30,125$ (E) $31,125$

16. Calculate the area of a triangle whose side lengths are $\sqrt{2}$, $3\sqrt{2}$ and $2\sqrt{5}$.

(A) 2 (B) $\sqrt{5}$ (C) 3 (D) 3.5 (E) $\sqrt{15}$

17. How many ordered pairs of positive integers (x, y) satisfy the equation $\dfrac{1}{x} - \dfrac{2}{y} = \dfrac{1}{6}$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

18. On a recent trip from Northburg to Southtown, Jill decided to make a detour so she could pass through Center City. Forty minutes after she left Northburg, she noted that the remaining distance to Center City was twice as much as what she had traveled so far. After traveling another twenty one miles, she calculated that the remaining distance to Southtown was twice as much as what she had left to get to Center City. She arrived in Southtown an hour and a half later. Assuming she traveled at a constant speed, how long was this trip from Northburg to Southtown?

(A) 99 miles (B) 108 miles (C) 112 miles (D) 127 miles (E) 142 miles

19. What is the sum of all two-digit numbers whose tens digit and units digit differ by exactly one?

(A) 878 (B) 890 (C) 900 (D) 990 (E) 991

20. How many real numbers differ by exactly 2 from their reciprocals?

(A) 0 (B) 2 (C) 3 (D) 4 (E) more than 4

21. Which of the following describes the set of points in the plane which satisfy the equation $y^2 - y - x^2y + x^2 = 0$?

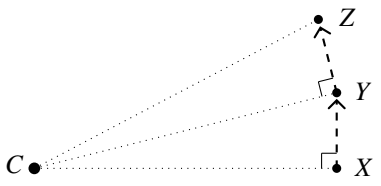
(A) the union of a parabola and a line (B) the union of two lines (C) a parabola
(D) a hyperbola (E) the union of a hyperbola and a line

22. Every Monday, Harvey puts a new puzzle on his blog. The puzzle for today goes like this: "My two sisters, all of my children and my younger brother and I were born between Jan. 1, 1901 and Dec. 31, 1999, each of us in a different year. Oddly, we all satisfy a very peculiar property. Each of us turned yx in some year $19xy$ where $0 \le y < x \le 9$ (a different year for each of us of course). And odder still, if someone born in $19ab$ satisfies this peculiar property, then one of us was born in that year. My sisters, my brother and I take care of the first four such years $19ab$, so how many pairs $0 \le y < x \le 9$ are there where my oldest child turned yx in the year $19xy$?"

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

23. Surveyors are laying out a rather unusual road through a park. The park is completely flat and forms a disc of radius 20 miles. From the center C, the road is to go exactly two miles north to a point B_0, then make a 90^o left turn and go another two miles to a point B_1. At B_1 the road turns left again (not nearly as sharply), this time perpendicular to $\overline{CB_1}$. As before, it goes exactly two miles in this direction to a point B_2. This pattern is followed for the entire road – at B_k, the road makes a left turn that is perpendicular to $\overline{CB_k}$ and goes exactly two miles in this direction to B_{k+1}. Eventually the road reaches a point A (one of the B_js) that is exactly 10 miles from C. Starting from A, how many **more** two-mile segments will be needed before the road gets out of the park? [In the figure, two consecutive segments are shown starting from an arbitrary point X to the point Y and then from Y to Z.]

(A) fewer than 30
(B) between 30 and 49
(C) between 50 and 69
(D) between 70 and 89
(E) at least 90



24. Consider the equation $\sqrt{x+3-4\sqrt{x-1}}+\sqrt{x+8-6\sqrt{x-1}}=1$ where x represents a real number. How many solutions are there?

(A) Exactly one solution (B) Exactly two solutions (C) Exactly three solutions
(D) Exactly four solutions (E) Infinitely many solutions

25. The Mainter brothers, Abe, Ben and Cal paint houses. They have been in the business so long that each knows exactly how many square feet he (and each of his brothers) paints in one hour and these rates never change. For their latest job, they calculated that if Abe and Ben did the job together, it would take exactly 11 hours. On the other hand, if Abe and Cal did the job together, it would take exactly 9 hours. Finally, Ben and Cal could do it in exactly 9.9 hours (or if you prefer, 9 hours and 54 minutes). They decided all three would paint this particular house. Ben and Cal started the job at 8 AM and Abe joined them at 9:00. Cal left at 1:30 and so Ben and Abe finished the job. After deducting the supply costs (paint etc.), the brothers split the net profit based on what percentage of the total square footage each painted. Who earned the most and who earned the least for this job?

(A) Abe the most, Ben the least (B) Abe the most, Cal the least
(C) Ben the most, Abe the least (D) Cal the most, Ben the least
(E) Cal the most, Abe the least