

1. Suppose f, g and h are polynomials of degrees 7, 8 and 9 respectively. Let d be the degree of the quotient $(f \circ g) \cdot h \div (f + g + h)$, where \circ means composition. Which of the following statements is true?

(A) $d \leq 40$ (B) $40 < d \leq 50$ (C) $50 < d \leq 60$ (D) $60 < d \leq 70$ (E) $d > 70$

2. A ball of radius 1 is rolling on the floor $Q: -3 \leq x \leq 3, -3 \leq y \leq 3$ of a square room, getting reflected elastically off its walls. Otherwise, it moves straight. Initially, it touches the Eastern wall of the room at the point $(3, 0)$, then moves in the North-West direction and hits the Northern wall at the point $(0, 3)$, then gets reflected and moves South-West, etc. Let L be the distance the ball travels before returning to the initial position. Which of the following statements is correct?

(A) $11 < L \leq 12$ (B) $12 < L \leq 14$ (C) $14 < L \leq 15$ (D) $15 < L \leq 16$ (E) $16 < L \leq 18$

3. There are exactly 2 positive values of r for which the system of equations

$$\begin{cases} x^2 + y^2 & = 9, \\ (x - 8)^2 + (y - 6)^2 & = r^2 \end{cases}$$

has a unique solution (x, y) . Let us denote these values of r by r_1 and r_2 , and put $s = r_1 + r_2$. Which of the following statements is true?

(A) $s \leq 10$ (B) $10 < s \leq 17$ (C) $17 < s \leq 21$ (D) $21 < s \leq 26$ (E) $s > 26$

4. What is the area of the region determined by $|x - 1| + |y - 1| \leq 2$?

(A) 2 (B) 4 (C) $4\sqrt{2}$ (D) 8 (E) $8\sqrt{2}$

5. Six points are distributed around a circle. In how many ways can you build two disjoint (i.e., non-intersecting) triangles using the six points as vertices?

(A) 1 (B) 2 (C) 3 (D) 4 (E) more than 4

6. The top of a rectangular box has area 40 square inches, the front has area 48 square inches, and the side has area 30 square inches. How high is the box?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

7. The lower two vertices of a square lie on the x -axis and the upper two vertices of the square lie on the parabola $y = 15 - x^2$. What is the area of the square?

(A) 9 (B) $10\sqrt{2}$ (C) 16 (D) 25 (E) 36

8. A math class has between 15 and 40 students. Exactly 25% of the class knows how to play poker. On a certain Wednesday, 3 students were absent (because they were participating in a math contest). On that day, exactly 20% of the students attending the class knew how to play poker. How many students attending the class on that day knew how to play poker?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

9. Pansies have five petals and lilacs have four petals. A bouquet has twenty flowers with a total of 92 petals. How many pansies does the bouquet have?
 (A) $3 \leq P \leq 7$ (B) $8 \leq P \leq 10$ (C) $11 \leq P \leq 14$ (D) $15 \leq P \leq 17$ (E) $P \geq 18$
10. A dictionary contains pages numbered 1 through 852. How many times does the number 8 appear as a digit in this numbering?
 (A) 85 (B) 146 (C) 165 (D) 217 (E) 218
11. Two married couples have purchased theater tickets and are seated in a row consisting of just four seats. If they take their seats in a completely random order, what is the probability that Jim and Paula (husband and wife) sit in the two seats on the far left?
 (A) $\frac{1}{12}$ (B) $\frac{1}{24}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) $\frac{1}{6}$
12. Inside a large circle of radius 10 are four small circles and one medium size circle as in the diagram below. The four small circles all have the same radius and each is tangent to the large circle and to two other small circles. The center of the medium circle is the same as the center of the large circle and the medium circle passes through the center of each small circle. What is the radius of the medium circle?
 (A) $10(2 - \sqrt{2})$
 (B) $5\sqrt{2}$
 (C) $3\sqrt{5}$
 (D) $10(\sqrt{3} - 1)$

in caterpillar population in the UNC Charlotte botanical gardens, population size (the carrying capacity), and $x = N/M$ is the relative size. Scientists have observed that if the population one spring has relative size x , then the population the next spring has relative size $4x(1 - x)$. There is at most one value x_0 such that if the relative size one spring is x_0 , then the relative size for the next spring is also x_0 . Which of the following statements is true?

- (A) $x_0 \leq 3/5$ (B) $x_0 \leq 3/5$ (C) $3/5 < x_0 \leq 7/8$ (D) $7/8 < x_0 < 1$ (E) no such x_0 exists

10. Find all positive integers with $a < b < c$ such that $1/a + 1/b + 1/c = 1$. What is the sum of all such $a + b + c$?

- (A) 10 (B) 11 (C) 12 (D) 13 (E) no such integers exist

15. One of the roots of the quadratic equation $x^2 - 9x + a = 0$ is twice the other root. Which of the following statements is true?
(A) $a \leq 5$ (B) $15 < a \leq 10$ (C) $10 < a \leq 15$ (D) $15 < a \leq 20$ (E) $a > 20$
16. In the quadratic equation $x^2 - 7x + a = 0$ the sum of the squares of the roots equals 39. Find a .
(A) 8 (B) 7 (C) 6 (D) 5 (E) 4
17. The sides of a right triangle form an arithmetic sequence, while their sum equals 48. Find the area of the triangle.
(A) 24 (B) 96 (C) 48 (D) 54 (E) 84
18. The ratio of the legs in a right triangle equals $3/2$, while the length of the hypotenuse is $\sqrt{52}$. Find the area of the triangle.
(A) 12 (B) 13 (C) 26 (D) 30 (E) 169
19. Fresh cucumbers are 90% water. It is known that in a week after they are picked the amount of water reduces to 80%. How much will 20 pounds of such cucumbers weigh after a week?
(A) 8 (B) 10 (C) 12 (D) 15 (E) 18
20. For how many pairs of digits (a, b) does $\sqrt{0.aaaaa\dots} = 0.bbbbb\dots$?
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5