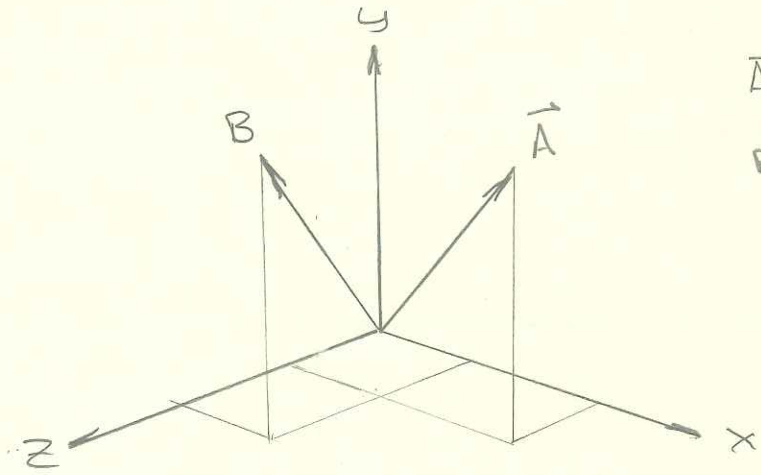
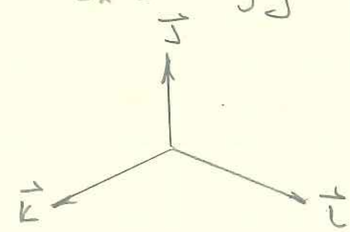
J. Hill 2010

VECTORS IN RECTANGULAR COORDINATES



$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$$

$$B = B_x \vec{i} + B_y \vec{j} + B_z \vec{k}$$



DOT PRODUCT $\vec{A} \cdot \vec{B}$

$$\begin{aligned}
\vec{A} \cdot \vec{B} &= (A_x \vec{i} + A_y \vec{j} + A_z \vec{k}) \cdot (B_x \vec{i} + B_y \vec{j} + B_z \vec{k}) \\
&= A_x B_x \vec{i} \cdot \vec{i} + A_x B_y \vec{i} \cdot \vec{j} + A_x B_z \vec{i} \cdot \vec{k} \\
&\quad + A_y B_x \vec{j} \cdot \vec{i} + A_y B_y \vec{j} \cdot \vec{j} + A_y B_z \vec{j} \cdot \vec{k} \\
&\quad + A_z B_x \vec{k} \cdot \vec{i} + A_z B_y \vec{k} \cdot \vec{j} + A_z B_z \vec{k} \cdot \vec{k}
\end{aligned}$$

BUT $\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1$

AND $\vec{i} \cdot \vec{j} = \vec{i} \cdot \vec{k} = \vec{j} \cdot \vec{i} = \vec{j} \cdot \vec{k} = \vec{k} \cdot \vec{i} = \vec{k} \cdot \vec{j} = 0$

SO $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$ (SCALAR)

CROSS PRODUCT $\vec{A} \times \vec{B}$

$$\begin{aligned}
\vec{A} \times \vec{B} &= (A_x \vec{i} + A_y \vec{j} + A_z \vec{k}) \times (B_x \vec{i} + B_y \vec{j} + B_z \vec{k}) \\
&= A_x B_x \vec{i} \times \vec{i} + A_x B_y \vec{i} \times \vec{j} + A_x B_z \vec{i} \times \vec{k} \\
&\quad + A_y B_x \vec{j} \times \vec{i} + A_y B_y \vec{j} \times \vec{j} + A_y B_z \vec{j} \times \vec{k} \\
&\quad + A_z B_x \vec{k} \times \vec{i} + A_z B_y \vec{k} \times \vec{j} + A_z B_z \vec{k} \times \vec{k}
\end{aligned}$$

BUT $\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$

AND $\vec{i} \times \vec{j} = k$, $\vec{j} \times \vec{k} = \vec{i}$, $\vec{k} \times \vec{i} = \vec{j}$, $\vec{j} \times \vec{i} = -\vec{k}$, $\vec{k} \times \vec{j} = -\vec{i}$
$\vec{i} \times \vec{k} = -\vec{j}$

SO $\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\vec{i} - (A_x B_z - B_x A_z)\vec{j} + (A_x B_y - A_y B_x)\vec{k}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$