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From Tds equations

$$s_2 - s_1 = \int_{T_1}^{T_2} c_p \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

For isentropic process, $s_2 - s_1 = 0$

$$\int_{T_{REF}}^{T} c_p \frac{dT}{T} = R \ln \frac{P}{P_{REF}}$$

where T_{REF} and P_{REF} are a reference state.

Define relative pressure: $P_R = \frac{P}{P_{REF}}$

$$P_{R_1} = \frac{P_1}{P_{REF}} , \quad P_{R_2} = \frac{P_2}{P_{REF}}$$

So $$\boxed{\frac{P_{R_2}}{P_{R_1}} = \frac{P_2}{P_1}}$$

and $\ln P_R = \frac{1}{R} \int_{T_{REF}}^{T} c_p \frac{dT}{T}$ ← Tabulated for various gasses

Define $v_R = \frac{RT}{P_R} \Rightarrow P_R = \frac{RT}{v_R}$

$$\frac{P_{R_1}}{P_{R_2}} = \frac{\frac{RT_1}{v_{r_1}}}{\frac{RT_2}{v_{R_2}}} = \frac{P_1}{P_2} = \frac{\frac{RT_1}{v_1}}{\frac{RT_2}{v_2}}$$

So $$\boxed{\frac{v_2}{v_1} = \frac{v_{R_2}}{v_{R_1}}}$$