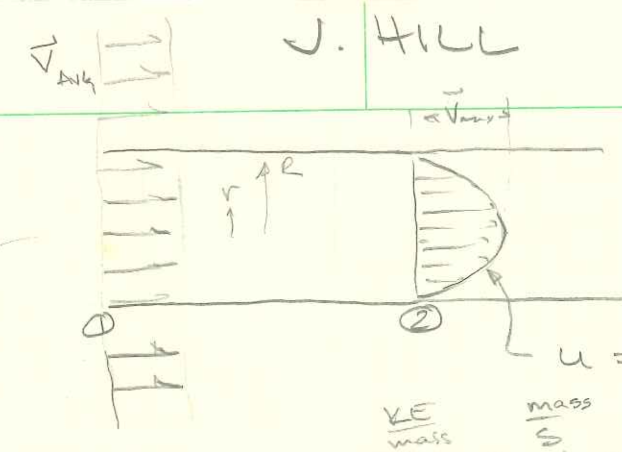
J. HILL

Look at correction for lam. KE/mass profile vs blunt



$$\frac{P}{\rho} + \boxed{\frac{\vec{V}^2}{2}} + gz = \text{const}$$

$$u = V_{max}\left[1 - \left(\frac{r}{R}\right)^2\right] = 2\vec{V}_{AVG}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

From earlier

$$\text{KE flux} = \int_A \underbrace{\frac{\vec{V}^2}{2}}_{\text{KE/mass}} \underbrace{\rho\, \vec{V}\cdot\vec{n}\, dA}_{\text{mass/s}}$$

$$\text{KE correction factor, } \alpha \equiv \frac{\left|\int_{A_2} \frac{\vec{V}^2}{2}\rho\, \vec{V}\cdot\vec{n}\, dA\right|}{\left|\int_{A_1} \frac{\vec{V}^2}{2}\rho\, \vec{V}\cdot\vec{n}\, dA\right|}$$

$$\alpha = \frac{1}{\rho A \vec{V}_{AVG}^3}\int \vec{V}^3 \rho\, dA = \frac{1}{A}\int\left(\frac{\vec{V}}{\vec{V}_{AVG}}\right)^3 dA = \frac{1}{\pi R^2}\int_{r=0}^{R}\left(\frac{\vec{V}}{\vec{V}_{AVG}}\right)^3 2\pi r\, dr$$

$$= 2\int_0^1\left(\frac{\vec{V}}{\vec{V}_{AVG}}\right)^3\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)$$

$$= 2(2)^3\int_0^1\left(\frac{\vec{V}}{\vec{V}_{max}}\right)\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)$$

$$= 16\int_0^1\left[1 - \left(\frac{r}{R}\right)^2\right]^3\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)$$

$$= 16\int_0^1\left[\frac{r}{R} - 3\left(\frac{r}{R}\right)^3 + 3\left(\frac{r}{R}\right)^5 - \left(\frac{r}{R}\right)^7\right] d\left(\frac{r}{R}\right)$$

$$= 16\left[\frac{1}{2}\left(\frac{r}{R}\right)^2 - \frac{3}{4}\left(\frac{r}{R}\right)^4 + \frac{3}{6}\left(\frac{r}{R}\right)^6 - \frac{1}{8}\left(\frac{r}{R}\right)^8\right]_0^1$$

$$= 16\left(\frac{1}{2} - \frac{3}{4} + \frac{1}{2} - \frac{1}{8}\right) = 2$$

$$\frac{P_1}{\rho} + \alpha\frac{\vec{V}^2}{2} + gz_1 = \text{const}$$

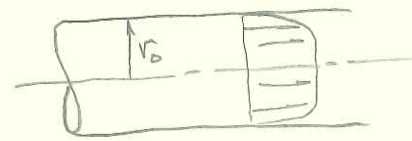
LESS IMPORTANT FOR TURBULENT FLOW SINCE VEL. PROFILE IS BLUNTED.

Turbulent Flow in Pipes

J. Hill Fall 10

Due to the chaotic nature of turbulent flow, there is no universal expression for the turbulent velocity profile for pipe flow. We must therefore rely on experimental data. The most common equation is

$$\frac{u}{u_{MAX}} = \left(1 - \frac{r}{r_o}\right)^{\frac{1}{n}}$$



where u is the time average of the fluctuating velocity and n is a function of Re_D. This equation does not apply very close to the wall and does not give zero velocity at pipe ℄.

Find $\frac{u_{AVG}}{u_{max}}$ as a function of n

$$u_{AVG} = \frac{Q}{A} = \frac{1}{\pi r_o^2}\int_0^{r_o} u\,\overbrace{(2\pi r\,dr)}^{dA}$$

$$= 2\,u_{MAX}\int_0^1 \left[1 - \frac{r}{r_o}\right]^{\frac{1}{n}}\left(\frac{r}{r_o}\right) d\left(\frac{r}{r_o}\right)$$

Let $1 - \frac{r}{r_o} = \eta$, so $\frac{r}{r_o} = 1 - \eta$, $d\left(\frac{r}{r_o}\right) = -d\eta$

then $$\frac{u_{AVG}}{u_{MAX}} = 2\int_{\eta=0}^{1} \eta^{\frac{1}{n}}(1-\eta)(-d\eta) = 2\int_0^1 \eta^{\frac{1}{n}}(1-\eta)\,d\eta$$

$$\frac{u_{AVG}}{u_{MAX}} = 2\left[\frac{1}{1+\frac{1}{n}}\eta^{1+n} - \frac{1}{2+\frac{1}{n}}\eta^{2+\frac{1}{n}}\right]_0^1 = 2\left[\frac{1}{1+\frac{1}{n}} - \frac{1}{2+\frac{1}{n}}\right]$$

$$= 2\left[\frac{n}{n+1} - \frac{n}{2n+1}\right] = 2\left\{\frac{n(2n+1) - n(n+1)}{(n+1)(2n+1)}\right\}$$

$$= 2\left[\frac{2n^2 + n - n^2 - n}{(n+1)(2n+1)}\right] = \frac{2n^2}{(n+1)(2n+1)}$$

From experiment, $n = 1.85 \log_{10} Re_{u_{max}} - 1.96$

$Re_{u_{max}}$	2×10^4	5×10^4	10^5	2×10^5	5×10^5	10^6	2×10^6
n	6	6.73	7.29	7.85	8.58	9.14	9.7
u_{AVG}/u_{max}	.791	.81	.823	.834	.846	.855	.862

If turbulent profile is integrated as we did for parabolic profile in laminar flow

$$\alpha = \left(\frac{u_{MAX}}{u_{AVG}}\right)^3 \frac{2n^2}{(3+n)(3+2n)}$$

n	α
6	1.08
10	1.03

since close to unity, $\alpha \approx 1$ is O.K. for turbulent flow