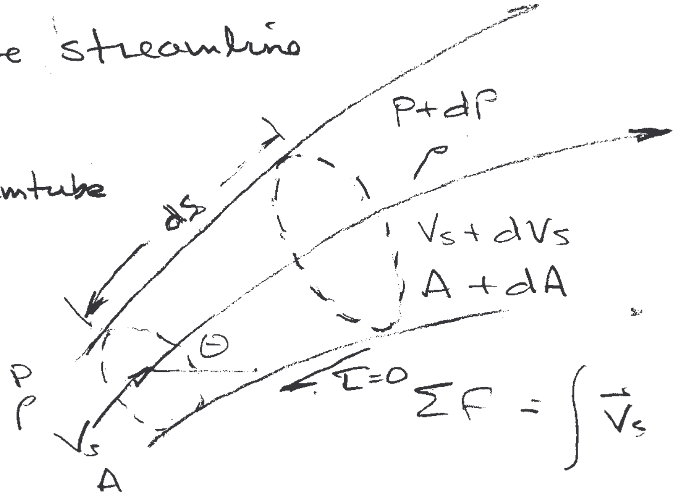
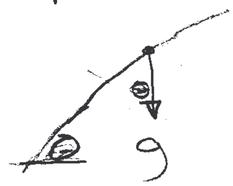
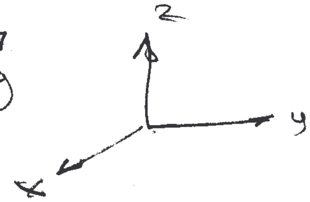
~~Apply momentum eq'n in the S (streamline)~~ direction for steady, incompressible, flow

Define streamline

Streamtube







$$\Sigma F = \int \vec{V}_s \, \rho \vec{V}\cdot\vec{n} \, dA \qquad (A+dA)-A$$

s-direction pressure forces

$$F_P = PA - (P+dP)(A+dA) + \left(P + \frac{dP}{2}\right) dA$$
$$= PA - PA - PdA - AdP - dPdA + PdA + \frac{dPdA}{2}$$
$$= -AdP - \frac{dPdA}{2}$$

s-direction body force

$$F_B = \rho g_s \, dV$$
$$= \rho(-g\sin\theta)\left(A + \frac{dA}{2}\right) ds$$

but $ds \sin\theta = dz$

$$F_B = -\rho g \left(A + \frac{dA}{2}\right) dz$$

s-dir. momentum flux

equal from cont.

$$\int \vec{V}_s \, \rho \vec{V}\cdot\vec{n} \, dA = \vec{V}_s\left[-\rho \vec{V}_s A\right] + (\vec{V}_s + dV_s)\left[\rho (V_s + dV_s)(A+dA)\right]$$
$$= \rho \vec{V}_s A \left[-\vec{V}_s + \vec{V}_s + dV_s\right]$$
$$= \rho \vec{V}_s A \, d\vec{V}_s$$

Substitute into momentum equation

$$-A\,dP - \frac{dP\,dA}{2} - \rho g A\,dz - \frac{\rho g\,dA\,dz}{2} = \rho \vec{V}_s A\, d\vec{V}_s$$

div. by ρA and drop H.O.T.

$$-\frac{dP}{\rho} - g\,dz = \vec{V}_s\, d\vec{V}_s$$

$$\frac{dP}{\rho} + g\,dZ + d\left(\frac{\vec{V}_s^2}{2}\right) = 0$$

Integrating

$$\boxed{\frac{P}{\rho} + gZ + \frac{\vec{V}_s^{\,2}}{2} = \text{const}}$$

Bernoulli's Eq'n

In general, different for different streamlines

1. Steady
2. frictionless
3. incompressible
4. along streamline
5. No work

~~Heat~~ ?

Define irrotational flow.

Or, from a different direction recall work eq'n for reversible, steady flow from thermo 1:

$$\cancel{w}^{\,0} = -\int v\,dP - \frac{\bar{V}_2^2 - \bar{V}_1^2}{2} - g(z_2 - z_1)$$

If $\rho = \text{const}$ and $w = 0$

$$-v(P_2 - P_1) - \frac{\bar{V}_2^2 - \bar{V}_1^2}{2} - g(z_2 - z_1)$$

$\rho = \frac{1}{v}$

$$\frac{P_1}{\rho} + \frac{\bar{V}_1^2}{2} + gZ_1 = \frac{P_2}{\rho} + \frac{\bar{V}_2^2}{2} + gZ_2$$