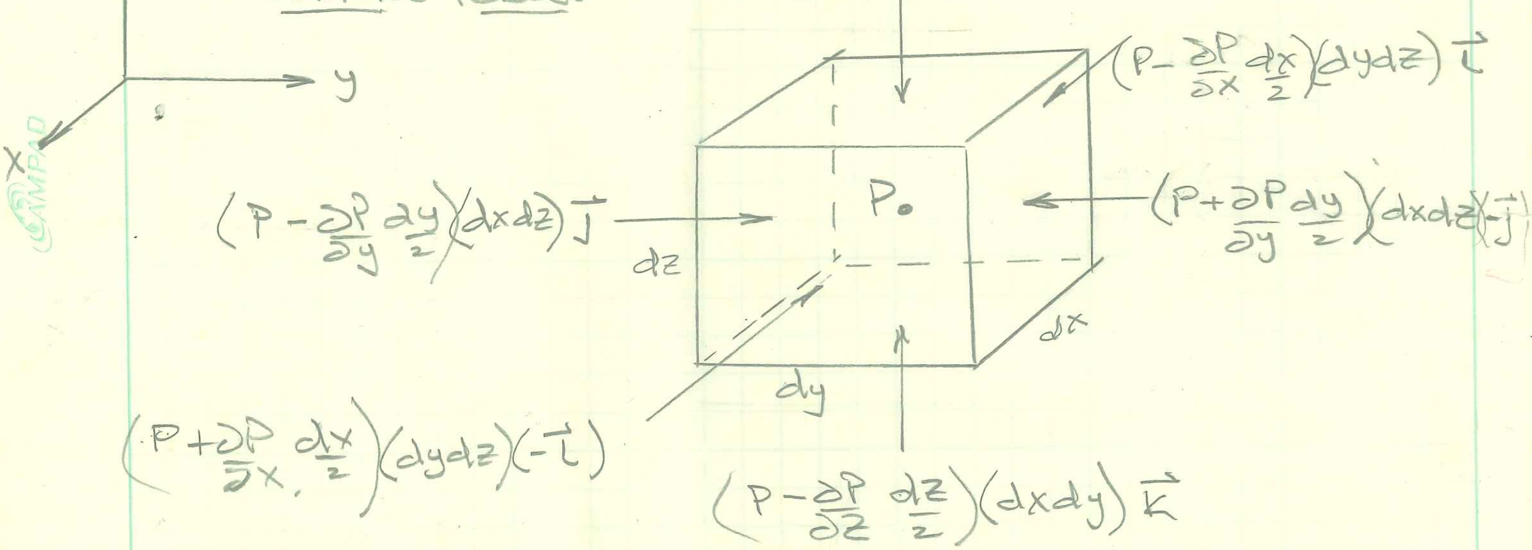
J. HILL 2018

In Statics we developed equations for the pressure distribution in a static, incompressible fluid. Now look at general case.

SURFACE FORCES



$$
\begin{aligned}
d\vec{F}_{SURF} &= \left(P-\frac{\partial P}{\partial x}\frac{dx}{2}\right)(dy\,dz)\vec{i} + \left(P+\frac{\partial P}{\partial x}\frac{dx}{2}\right)(dy\,dz)(-\vec{i}) \\
&+ \left(P-\frac{\partial P}{\partial y}\frac{dy}{2}\right)(dx\,dz)\vec{j} + \left(P+\frac{\partial P}{\partial y}\frac{dy}{2}\right)(dx\,dz)(-\vec{j}) \\
&+ \left(P-\frac{\partial P}{\partial z}\frac{dz}{2}\right)(dx\,dy)\vec{k} + \left(P+\frac{\partial P}{\partial z}\frac{dz}{2}\right)(dx\,dy)(-\vec{k})
\end{aligned}
$$

COLLECTING TERMS

$$
\begin{aligned}
d\vec{F}_{SURF} &= -\left(\frac{\partial P}{\partial x}\vec{i} + \frac{\partial P}{\partial y}\vec{j} + \frac{\partial P}{\partial z}\vec{k}\right)dx\,dy\,dz \\
&= -\nabla P \underbrace{dx\,dy\,dz}_{dV}
\end{aligned}
$$

BODY FORCES $\quad d\vec{F}_B = \vec{g}\,dm = \vec{g}\rho\,dV = \vec{g}\rho\,dx\,dy\,dz$

$$d\vec{F} = d\vec{F}_{SURF} + d\vec{F}_{BODY} = -\nabla P\,dx\,dy\,dz + g\rho\,dx\,dy\,dz$$

$$\boxed{\frac{d\vec{F}}{dV} = -\nabla P + \rho\vec{g}}$$

For a fluid particle

$$d\vec{F} = \vec{a}\, dm = \vec{a} \rho\, dV$$

so $\dfrac{d\vec{F}}{dV} = \boxed{\vec{a}\rho = -\nabla P + \rho \vec{g}}$ (from previous page)

Eqn 2.2 7$^{\text{TH}}$ ed

for a static fluid $a = 0$

so $-\nabla P + \rho \vec{g} = 0$ (sign diff. from book since we haven't given a direction for g.)

Pressure forces = body forces

$$\rho a_x = -\frac{\partial P}{\partial x} + \rho g_x = 0$$

$$\rho a_y = -\frac{\partial P}{\partial y} + \rho g_y = 0$$

$$\rho a_z = -\frac{\partial P}{\partial z} + \rho g_z = 0$$

For static fluid with z-axis vertical, $g_z = -g\vec{k}$

$$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial y} = 0, \quad \frac{dP}{dz} = -\rho g$$

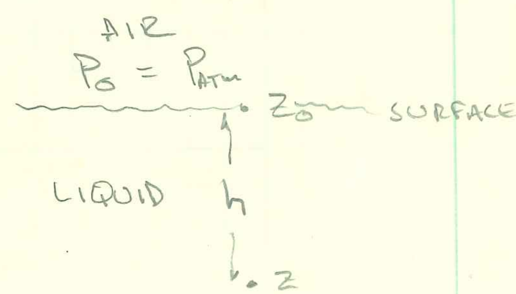
$$\boxed{P_2 - P_1 = -\int_1^2 \rho g \, dz}$$

Valid when

1. Fluid is static
2. Gravity is only body force
3. z-axis vertical

If variation in g is neglible and density is constant

$$P - P_o = \rho g (z_o - z)$$



$$\boxed{P = P_{ATM} + \rho g h}$$