Heat Transfer Equation Sheet

Conduction: $q_x = -kA\frac{dT}{dx}$

Convection: $q = hA(T_s - T_\infty)$

Radiation: $q_r = \varepsilon A\sigma T^4$

Heat Exchangers: $\Delta T_{LM} = \{(T_{Ho} - T_{Co}) - (T_{Hi} - T_{Ci})\}/\{\ln[(T_{Ho} - T_{Co})/(T_{Hi} - T_{Ci})]\}$ Parallel flow

$q = UA\,\Delta T_{LM}$ $\Delta T_{LM} = \{(T_{Ho} - T_{Ci}) - (T_{Hi} - T_{Co})\}/\{\ln[(T_{Ho} - T_{Ci})/(T_{Hi} - T_{Co})]\}$ Counter flow

$NTU = UA/C_{min}$ $C_r = C_{min}/C_{max}$ $\varepsilon = q/q_{max}$ $q_{max} = C_{min}(T_{Hi} - T_{Ci})$

Diffusion equations:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$ uniform properties, $\alpha = \frac{k}{\rho C_p}$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(\frac{\partial T}{\partial \phi}\right) + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$ Cylindrical coordinates

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Ohms Law Analogy, $q = \frac{\Delta T}{R_t}$

Plane wall: $R_{t,cond} = \frac{L}{kA}$, $R_{t,conv} = \frac{1}{hA}$

Cylinder: $R_{t,cond} = \frac{\ln(r_2/r_1)}{2\pi Lk}$, $R_{t,conv} = \frac{1}{2\pi rLh}$

Generation: Plane wall, $T_s = T_\infty + \dot{q}L/h$ $T_o = T_s + \dot{q}L^2/(2k)$ $\frac{T - T_o}{T_s - T_o} = (x/L)^2$

Cylinder, $T(r) = \frac{\dot{q}r_o^2}{4k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s$ $T_s = T_\infty + \frac{\dot{q}r_o}{2h}$

$\frac{T - T_s}{T_o - T_s} = 1 - \left(\frac{r}{r_o}\right)^2$

Fins of uniform cross section: $\theta = T - T_\infty$ $m^2 = hP/kA_c$ $M = (hPkA_c)^{1/2}\theta_b$

Infinite fin: $\theta = \theta_b e^{-mx}$ $q_f = M$

Adiabatic tip: $\theta = \theta_b\frac{\cosh[m(L-x)]}{\cosh(mL)}$ $q_f = M\tanh(mL)$

Prescribed tip: $\theta = \theta_b\frac{(\theta_L/\theta_b)\sinh(mx) + \sinh[m(L-x)]}{\sinh(mL)}$ $q_f = M\frac{\cosh(mL) - (\theta_L/\theta_b)}{\sinh(mL)}$

Active tip: $\theta = \theta_b\frac{(h/mk)\sinh[m(L-x)] + \cosh[m(L-x)]}{(h/mk)\sinh(mL) + \cosh(mL)}$ $q_f = M\frac{\sinh(mL) + (h/mk)\cosh(mL)}{\cosh(mL) + (h/mk)\sinh(mL)}$

All fins: Effectiveness, $\varepsilon_f = \frac{q_f}{hA_b\theta_b}$ Efficiency, $\eta_f = \frac{q_f}{hA_f\theta_b}$

Lumped Capacitance: $Bi = hL_c/k < 0.1$ $Fo = \alpha t/(L_c)^2$ $\frac{T - T_\infty}{T_i - T_\infty} = e^{-t/\tau_t}$ $\tau_t = \frac{\rho VC_p}{hA_s}$ $Q = \rho VC_p\theta_i(1 - e^{-t/\tau_t})$

Semi-infinite solid:

imposed temperature: $\frac{T - T_s}{T_i - T_s} = \mathrm{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$

Finite difference methods:

Explicit, 1-D unsteady: $T_m^{p+1} = Fo(T_{m+1}^p + T_{m-1}^p) + (1 - 2Fo)\,T_m^p$ $Fo = \alpha\Delta t/(\Delta x)^2 \le 1/2$ for stability

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Convection, Steady flow over a flat plate: $Pr = C_p\mu/k = \mu/(\rho\alpha)$ $Re_x = \rho u_\infty x/\mu$ $Nu_x = hx/k_f$

Laminar: $\delta = 5x\,Re_x^{-1/2}$ $C_{f,x} = 0.664\,Re_x^{-1/2}$ $\overline{C_{f,L}} = 1.328\,Re_L^{-1/2}$ $Nu_x = 0.332\,Re_x^{1/2}\,Pr^{1/3}$ $\overline{Nu_L} = 0.664\,Re_L^{1/2}\,Pr^{1/3}$

Mixed laminar and turbulent for $Re_{x,c} < Re_x < 10^8$: $\delta = 0.37x\,Re_x^{-1/5}$ $C_{f,x} = 0.0592\,Re_x^{-1/5}$ $Nu_x = 0.0296\,Re_x^{4/5}\,Pr^{1/3}$

Mixed laminar and turbulent with $Re_{x,c} = 5\times10^5$: $\overline{C_{f,L}} = 0.074\,Re_L^{-1/5} - 1742/Re_L$ $\overline{Nu} = (0.037Re_L^{4/5} - 871)Pr^{1/3}$

Note: If the laminar region is small, the constants 1742 and 871 in the above equations become zero.