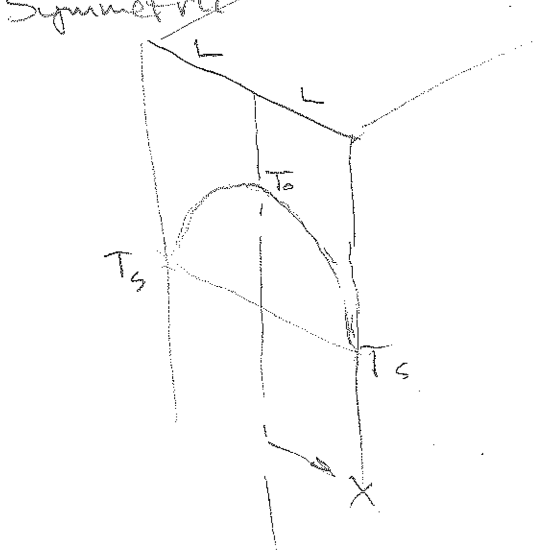
J. HILL



Plane wall with uniform distribution of heat sources

Assume 1-d, uniform prop., steady

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

B.C. @ $x = 0$, $\frac{dT}{dx} = 0$
$T = T_o$
@ $x = \pm L$ $T = T_s$

$$\frac{dT}{dx} = -\frac{\dot{q}}{k}x + C_1 \Rightarrow C_1 = 0$$

$$T = -\frac{\dot{q}}{k}\frac{x^2}{2} + C_2 \Rightarrow C_2 = T_o$$

$$\left.\begin{aligned} T - T_o &= -\frac{\dot{q}}{2k}x^2 \quad (1) \\ T_s - T_o &= -\frac{\dot{q}}{2k}L^2 \end{aligned}\right\} \quad \frac{T - T_o}{T_s - T_o} = \left(\frac{x}{L}\right)^2 \text{ parabolic} \quad (2)$$

To get mid-plane temp. use energy balance

$$q_{CONV} = q_{GEN}$$

$$2\left[-kA\left.\frac{dT}{dx}\right)_{x=L}\right] = \dot{q}A\,2L \quad (3)$$

from (2) $\left.\frac{dT}{dx}\right)_{x=L} = (T_s - T_o)\left.\frac{2x}{L^2}\right)_{x=L} = (T_s - T_o)\frac{2}{L}$

from (3) $-k(T_s - T_o)\frac{2}{L} = \dot{q}L$

$$\boxed{T_o = T_s + \frac{\dot{q}L^2}{2k}}$$

from (1) $\left.\frac{dT}{dx}\right|_{x=L} = -\frac{\dot{q}x}{k} = -\frac{\dot{q}L}{k}$ (4)

At surface

$$-k\left.\frac{dT}{dx}\right|_{x=L} = h(T_s - T_\infty)$$

$$\dot{q}L = h(T_s - T_\infty)$$

$$\boxed{T_s = T_\infty + \frac{\dot{q}L}{h}} \quad 3.46$$