

# Karnaugh-map usage

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Plot 1s corresponding to minterms of function.

Circle largest possible rectangular sets of 1s.

- # of 1s in set must be power of 2
- OK to cross edges

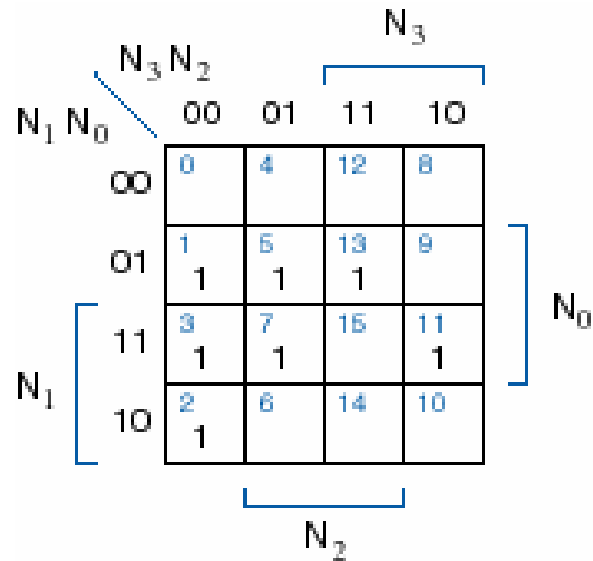
Read off product terms, one per circled set.

- Variable is 1  $\implies$  include variable
- Variable is 0  $\implies$  include complement of variable
- Variable is both 0 and 1  $\implies$  variable not included

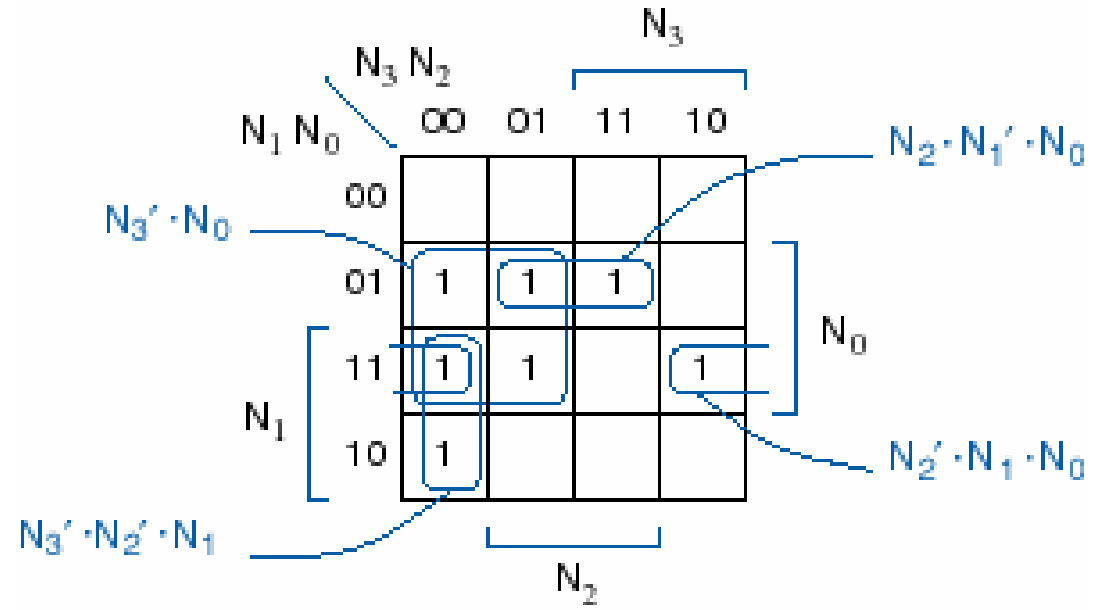
Circled sets and corresponding product terms are called  
“prime implicants”

Minimum number of gates and gate inputs

# Prime-number detector

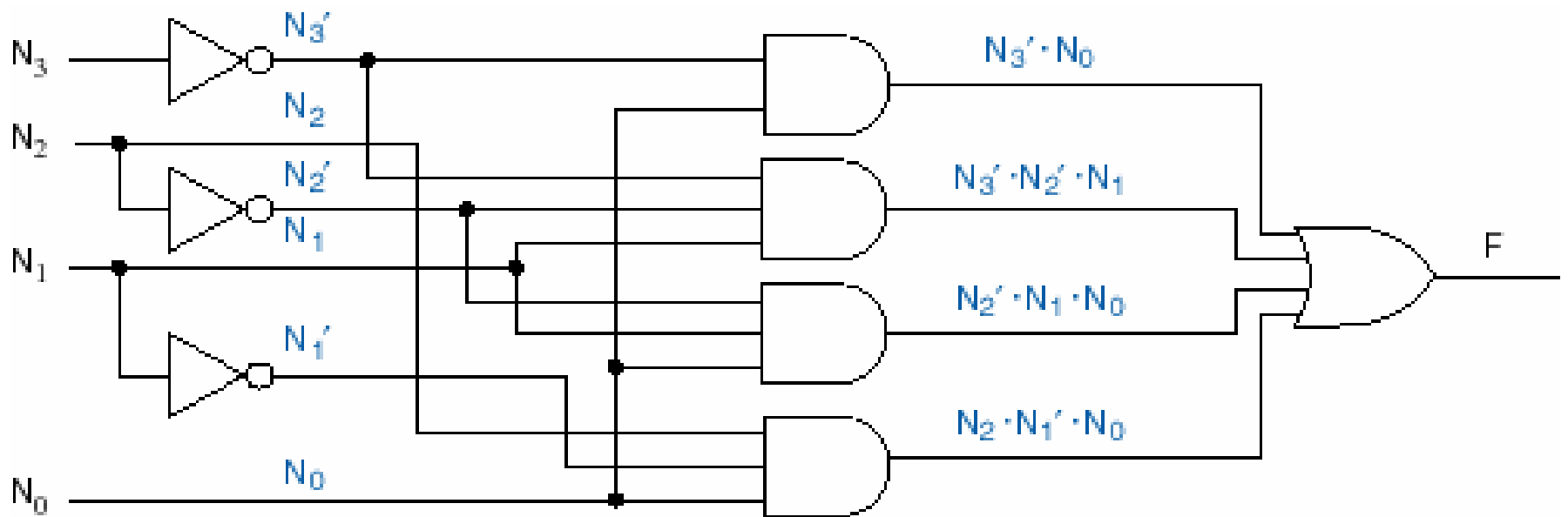


$$F = \sum_{N_3, N_2, N_1, N_0} (1, 2, 3, 5, 7, 11, 13)$$



$$F = N_3' \cdot N_0 + N_3' \cdot N_2' \cdot N_1 + N_2' \cdot N_1 \cdot N_0 + N_2 \cdot N_1' \cdot N_0$$

## Resulting Circuit.



# Another example

		W X		W	
		00	01	11	10
Y Z	00	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10

X

Z

$$F = \Sigma_{W,X,Y,Z}(5,7,12,13,14,15)$$

		W X		W	
		00	01	11	10
Y Z	00			1	
	01		1	1	
	11		1	1	
	10			1	

X

Z

$$F = X \cdot Z + W \cdot X$$

# Yet another example

		W X		W	
		00	01	11	10
Y Z	00	0	4 1	12 1	8
	01	1 1	5 1	13 1	9 1
	11	3 1	7	15 1	11 1
	10	2	6	14 1	10

X

Z

$$F = \sum_{W,X,Y,Z}(1,3,4,5,9,11,12,13,14,15)$$

		W X		W	
		00	01	11	10
Y Z	00		1	1	
	01	1	1	1	1
	11	1		1	1
	10			1	

X

Z

$X \cdot Y'$   
 $W \cdot Z$   
 $Y' \cdot Z$   
 $X' \cdot Z$   
 $W \cdot X$

$$F = X \cdot Y' + X' \cdot Z + W \cdot X$$

Distinguished 1 cells  
Essential prime implicants

## Another Example

$$F(W,X,Y,Z) = \sum m(0,1,2,4,5,6,8,9,12,13,14)$$

W X		W		
		11	10	
Y Z	00	01	11	10
	00	0	4	12
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Diagram illustrating a 4-variable Karnaugh map for the function  $F(W,X,Y,Z) = \sum m(0,1,2,4,5,6,8,9,12,13,14)$ . The map is a 4x4 grid with variables W, X, Y, and Z. The top row is labeled W X, and the columns are labeled W (11, 10) and Z (00, 01, 11, 10). The left side is labeled Y Z, and the rows are labeled Y (00, 01, 11, 10). The cells contain minterm numbers from 0 to 15. Blue brackets highlight the minterms 0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14, which correspond to the function's output.

## Another Example

$$F(W,X,Y,Z) = \sum m(0,1,2,3,6,8,9,10,11,14)$$

W X		W			
		11	10		
Y Z	00	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10

# Another Example

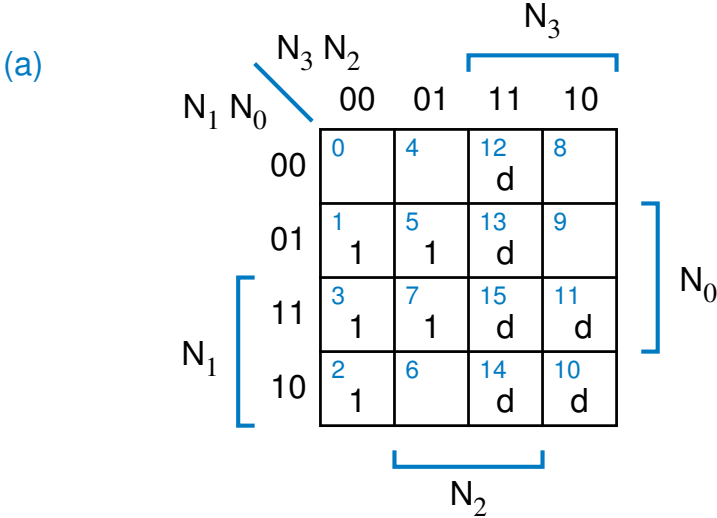
$$F(W,X,Y,Z) = \sum m(\quad)$$

		W X		W	
		00	01	11	10
Y Z	00	0	4	12	8
	01	1	5	13	9
Y	11	3	7	15	11
	10	2	6	14	10

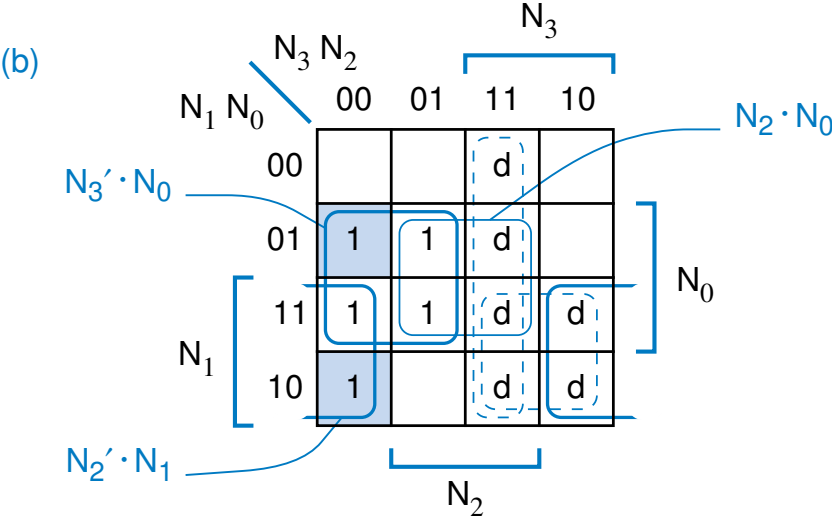


# Don't Cares

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 Digital Design Principles and Practices, 3/e



$$F = \sum_{N_3, N_2, N_1, N_0} (1, 2, 3, 5, 7) + d(10, 11, 12, 13, 14, 15)$$



$$F = N_3' \cdot N_0 + N_2' \cdot N_1$$

## Another Example

$$F(W,X,Y,Z) = \sum m(0,1,2,3,6,8,9,10,11,14) + d(7,15)$$

Y Z		W X		W	
		00	01	11	10
Y	00	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10

Diagram illustrating a 4x4 Karnaugh map for the function  $F(W,X,Y,Z)$ . The map is labeled with variables W, X, Y, and Z. The top row is labeled W X, and the columns are labeled W X. The left side is labeled Y Z, and the rows are labeled Y Z. The map contains the following values:

- Row 00: 0, 4, 12, 8
- Row 01: 1, 5, 13, 9
- Row 11: 3, 7, 15, 11
- Row 10: 2, 6, 14, 10

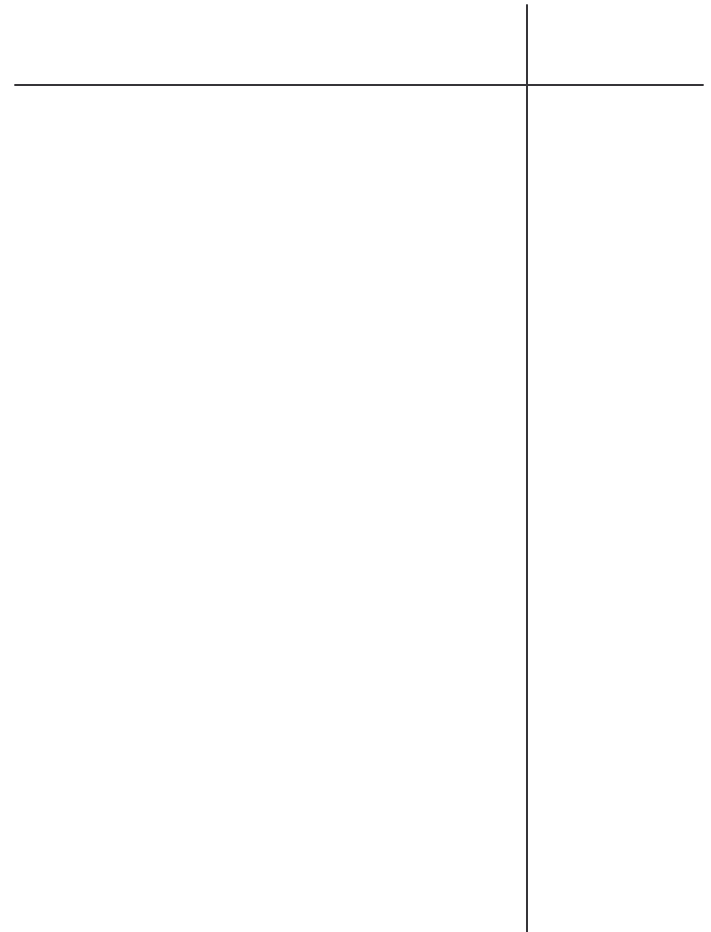
Blue brackets indicate groupings: a horizontal bracket above the columns 11 and 10 labeled W; a vertical bracket to the right of the rows 01 and 11 labeled Z; and a horizontal bracket below the columns 00 and 01 labeled X. A vertical bracket to the left of the rows 11 and 10 is labeled Y.

# Another Example

$$F(W,X,Y,Z) = \sum m(\quad) + d(\quad)$$

		W X		W	
		00	01	11	10
Y Z	00	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10

Diagram annotations: A blue bracket labeled 'Y' spans the rows 11 and 10. A blue bracket labeled 'Z' spans the rows 01 and 11. A blue bracket labeled 'X' spans the columns 01 and 11. A blue bracket labeled 'W' spans the columns 11 and 10.



# Resulting Circuit

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$$F(W,X,Y,Z) = \sum m(\quad) + d(\quad)$$

# Another Example

$$F(V,W,X,Y,Z) = \sum m(0,1,2,3,16,17,18,19,20,21,22) + d(23,30,31)$$

Truth table for  $V=0$ :

		W X		W	
		00	01	11	10
Y Z	00	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10

Labels:  $V=0$ ,  $X$ ,  $Z$ ,  $Y$

Truth table for  $V=1$ :

		W X		W	
		00	01	11	10
Y Z	00	16	20	28	24
	01	17	21	29	25
	11	19	23	31	27
	10	18	22	30	26

Labels:  $V=1$ ,  $X$ ,  $Z$ ,  $Y$

## Resulting Circuit

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$$F(V,W,X,Y,Z) = \sum m(0,1,2,3,16,17,18,19,20,21,22) + d(23,30,31)$$

# Another Example

$$F(V,W,X,Y,Z) = \Sigma m(\quad) + d(\quad)$$

Truth table for  $V=0$ :

		W X		W	
		00	01	11	10
Y Z	00	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10

Labels:  $V=0$ ,  $X$ ,  $Z$ ,  $Y$

Truth table for  $V=1$ :

		W X		W	
		00	01	11	10
Y Z	00	16	20	28	24
	01	17	21	29	25
	11	19	23	31	27
	10	18	22	30	26

Labels:  $V=1$ ,  $X$ ,  $Z$ ,  $Y$

# Current Logic Design

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Lots more than 6 inputs -- can't use Karnaugh maps

Use software to synthesize logic expressions and minimize logic

Hardware Description Languages -- VHDL and Verilog