10/15/2009 Outline

- Friday 10/16/2009 Recitation starts at 8 AM for Tutorial on Tools and SW
- Exam 1 handed back on Friday
- HW 6 Due today
- HW 7 Due ~~10/22 Thursday~~ Friday 10/23

Chapter 4 - Digital Design

Adder:

$$\begin{array}{r} 01 \\ +\ 10 \\ \hline 11 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\ 1}\ \ \\ 01 \\ +\ 11 \\ \hline 100 \end{array}$$

2 bit adder $2^{(2+2)} = 16$ rows

8 bit adder $2^{(8+8)} = 65,536$ Rows

16 bit addr $2^{(16+16)} = \sim 4$ Billion

Add 1 column at a time (like we humans do)

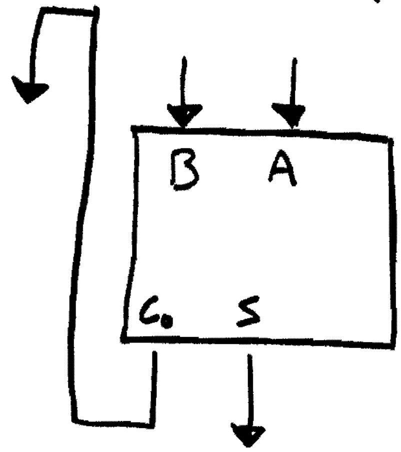
$$\begin{array}{r} {\scriptstyle 1\ 1\ 1\ 1}\ \ \\ 1\ 1\ 1\ 1 \\ +\ 0\ 1\ 1\ 0 \\ \hline 0\ 1 \end{array}$$

etc

[redacted]

Half Adder:

(1-bit Adder
sum and carry out)

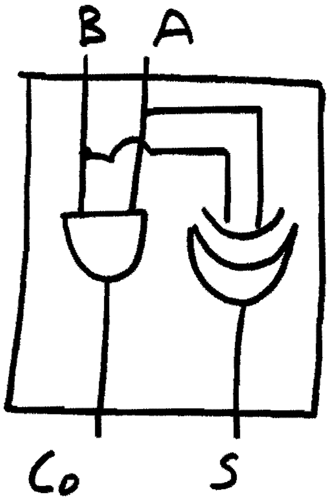


Capture the function

A	B	Co	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$C_o = A \cdot B$

$S = A \text{ XOR } B = (A'B + AB')$

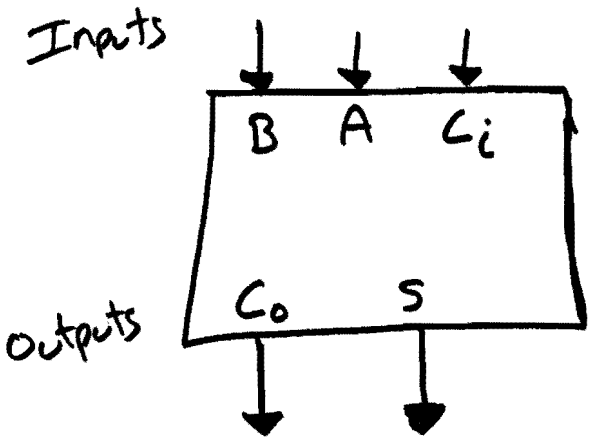


Limitation: multiple bit Half Adder does not allow us to consider Carry out

```
   1 ← how to add 3 inputs?
1011
1001
----
   ?
```

Full Adder

Adds 3 bits



Inputs Outputs

A	B	C_i	C_o	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$$C_o = \bar{a}bc + a\bar{b}c + ab\bar{c} + abc$$

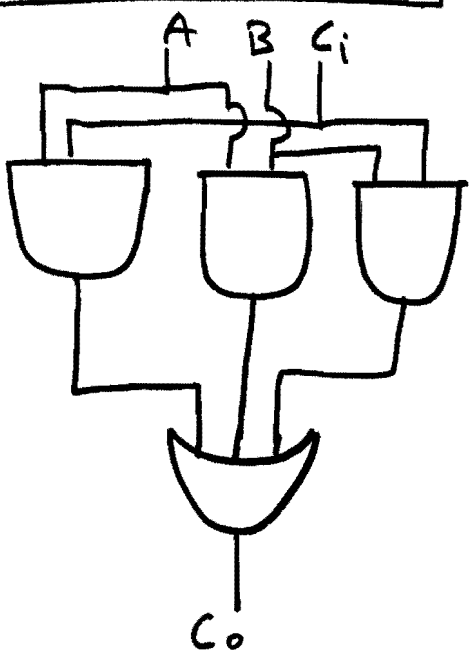
$$C_o = bc(\bar{a}+a) + a\bar{b}c + ab\bar{c}$$

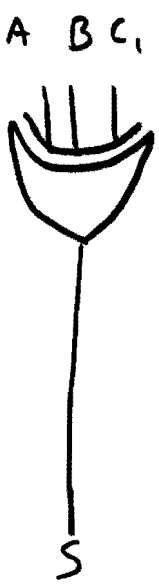
$$C_o = bc + a(\bar{b}c + b\bar{c})$$

$$bc + a(c + b)$$

$$\boxed{C_o = bc_i + ac_i + ab}$$

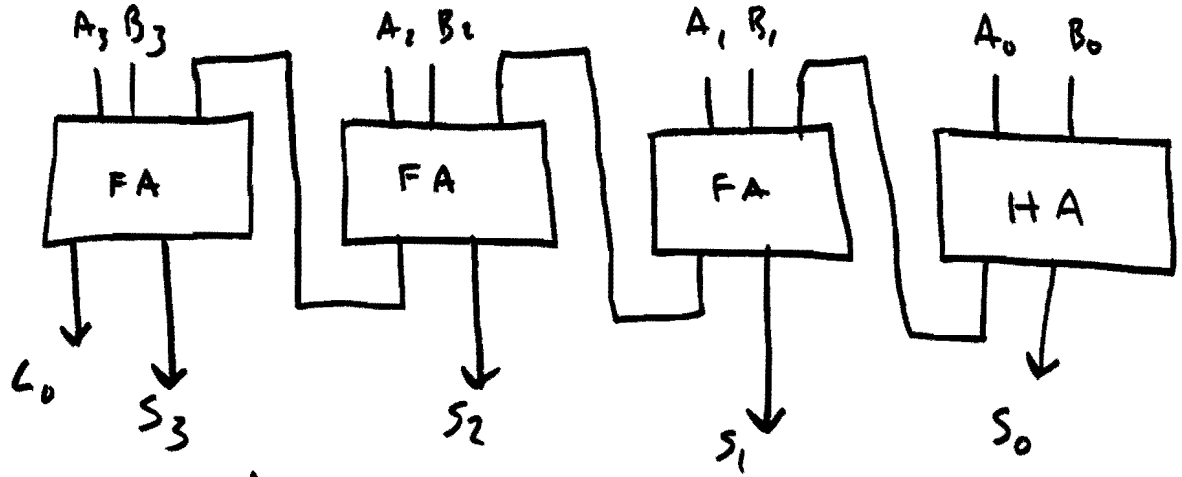
$$\boxed{S = a \text{ xor } b \text{ xor } c_i}$$

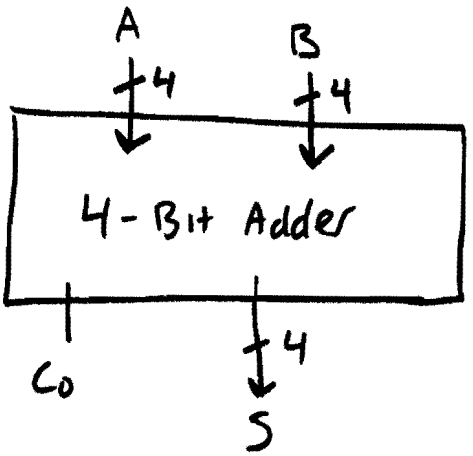




Carry-Ripple Adder V1.0

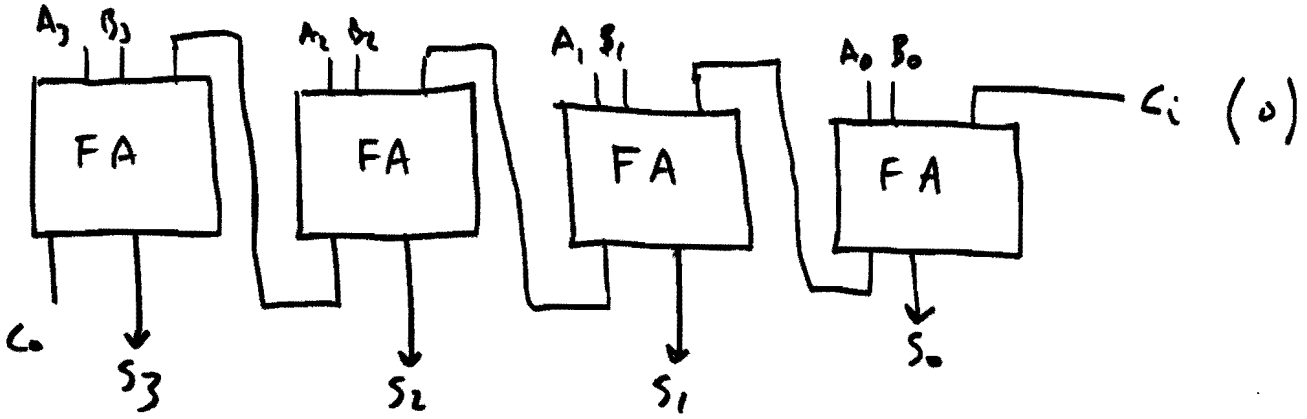
Multiple Bit Adder





Carry-Ripple Adder V2.0

4-Full Adders



Magnitude Comparator

A = 1011

Consider: B = 1001

equal

unequal

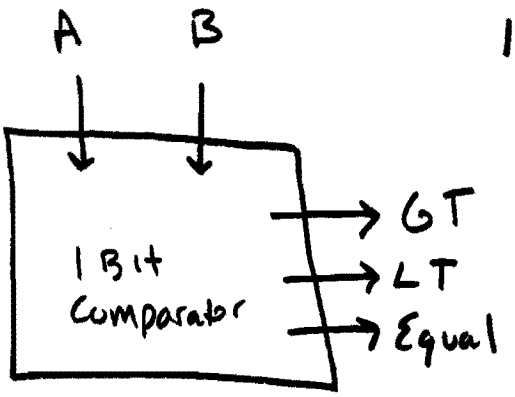
A > B

1 > 0

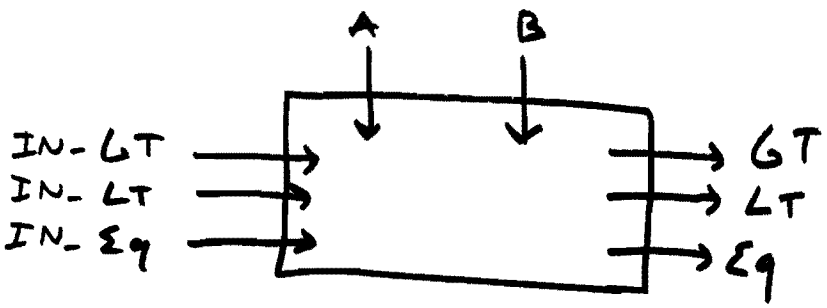
A = 1011

B = 1011

A = B



How to Chain multiple together?



Initial input is ~~A=B~~ Eq = 1

Comparator Greater Than? $A > B$

$$OUT\_GT = IN\_GT + (IN\_EQ \cdot A \cdot B')$$

$A > B$ $\quad A = 1, B = 0$

Comparator Less Than? $A < B$

$$OUT\_LT = IN\_LT + (IN\_EQ \cdot A' \cdot B)$$

$A < B$ $\quad A = 0 \quad B = 1$

Already know how to do an equality comparator ✓

$$OUT\_EQ = IN\_EQ \cdot (A \text{ XOR } B)$$

Worst Case Propagation Delay?

AND, OR, NOT = $1 T_p$

XOR = $2 T_p$

1 cell (1 Bit comparator): Equality uses 1 XOR, 1 AND

$$2 T_p + 1 T_p = 3 T_p$$

$$3 T_p + 2 \cdot (1 T_p) + 2 T_p$$

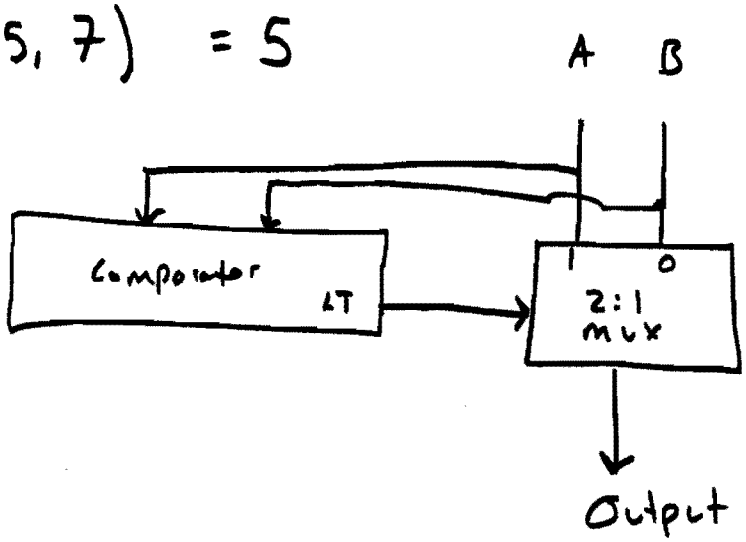
1st cell — middle cells — last cell

> or <

Minimum of Two Numbers

MIN (A, B)

MIN (5, 7) = 5



A < B

if yes then
mux select bit = 1
So output = A
else
output = B

Multiplier Slide 29

$$\begin{array}{r} 6 \\ \times\ 3 \\ \hline 18 \end{array} \qquad \begin{array}{r} 0110 \\ \times\ 0011 \\ \hline 0110 \\ 0110 \\ 0000 \\ 0000 \\ \hline 00100110 \end{array}$$

AND

Then Add together

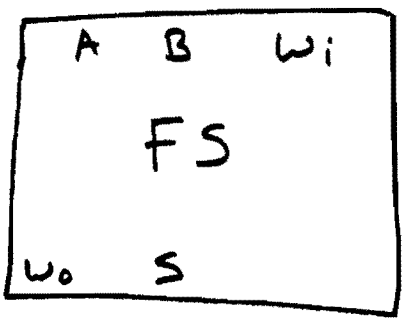
$$10010_2 = 16 + 2 = \boxed{18}$$

A_3	A_2	A_1	A_0
B_3	B_2	B_1	B_0
$A_3 \cdot B_0$	$A_2 \cdot B_0$	$A_1 \cdot B_0$	$A_0 \cdot B_0$
$A_3 \cdot B_1$	$A_2 \cdot B_1$	$A_1 \cdot B_1$	$A_0 \cdot B_1$	0
⋮			⋮

AND GATES Feeding into Adders

Subtractor

Not using 2's complement



w_1 is borrowed from Right
w_0 borrowed on left

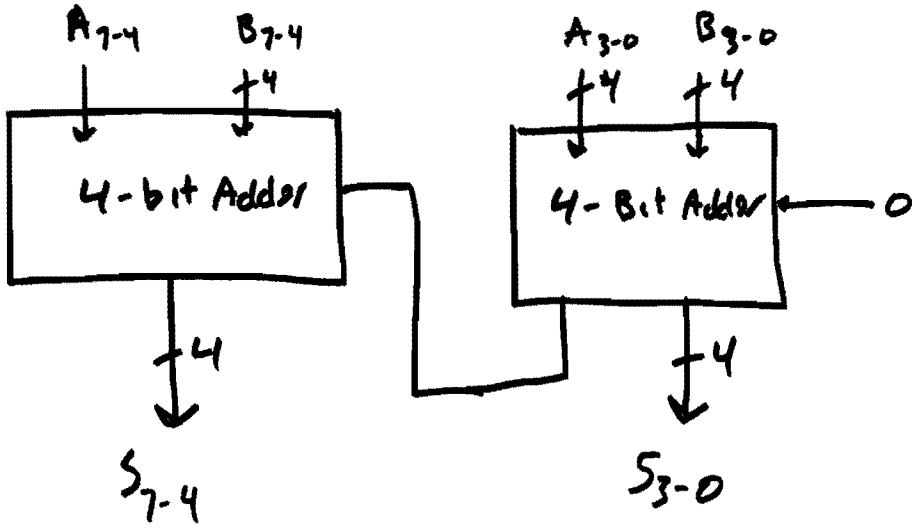
A	B	w_i	w_o	S
0	0	0	0	0
0	0	1	0	0
0	1	0	1	0
0	1	1	0	0
1	0	0	0	1
1	0	1	0	1
1	1	0	0	0
1	1	1	0	0

2's complement Adder

A + B is Adder

A - B is A + (- B)

Cascading Adders



Slides from Ch. 4 (up to 17)

Comparators:

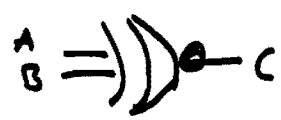
Equality Comparator

A = B iff A=0 & B=0
or
A=1 & B=1

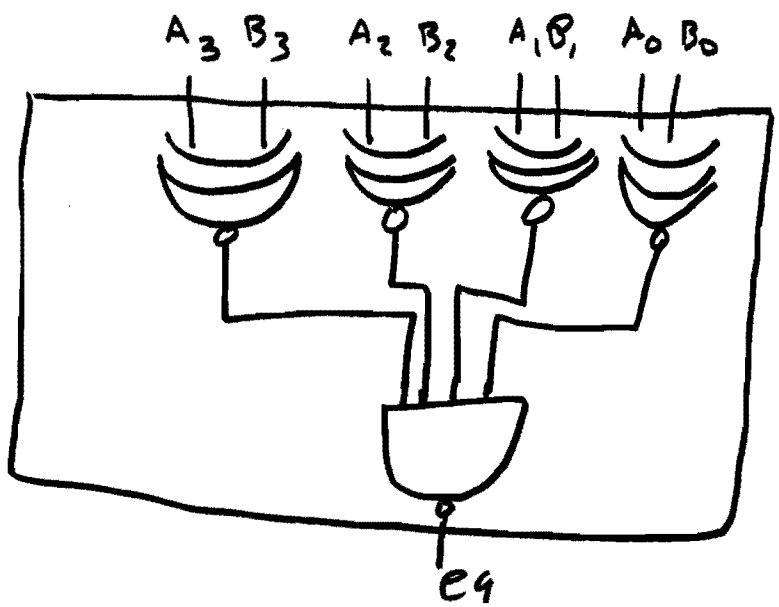
What Gate does this?

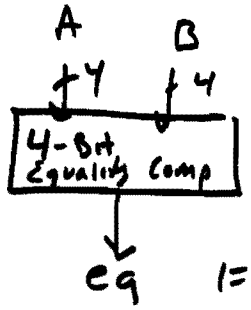
A	B	C
0	0	1
0	1	0
1	0	0
1	1	1

XNOR



4 Bit Equality Comparator?





1 = Yes
0 = No