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The Application of Elliptic Curves Cryptography in Embedded Systems

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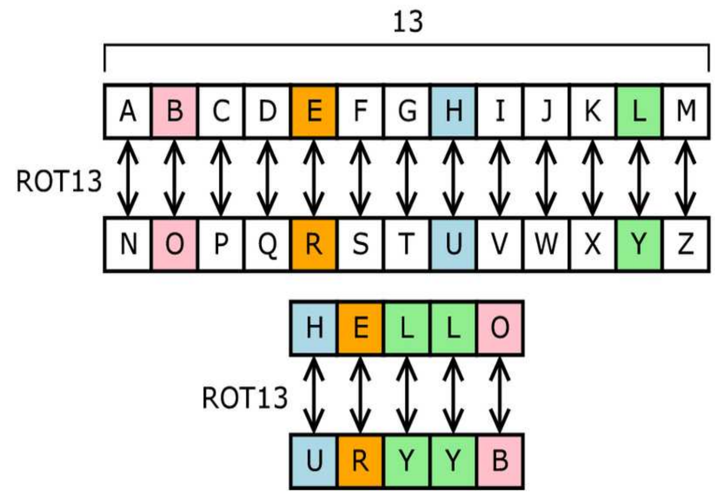
Introduction to Cryptography

Components of a Cryptosystem:

a. Plain text
b. Cipher
c. Code
d. Key

Popular Schemes used:

a. Public-key cryptography
   - Message is encrypted using a public key.
   - It is decrypted using a private key.
   - Private key is related to public key.
b. Three pass protocol
   - Message is encrypted by sender.
   - Message is super encrypted by receiver.
   - Sender decrypts message using private key.
   - Key exchange is not required.



Source:
http://upload.wikimedia.org/wikipedia/commons/2/2a/ROT13.png

Strategies for Public key Infrastructure

- RSA (Ron Rivest, Adi Shamir and Len Adleman)
  - Product of two large prime numbers is used to create a public key and private key.
  - With suitably large prime numbers, the problem of factorization increases.
  - Key sizes increase as the need of security level increase.

- DSA (Digital Signal Algorithm)
  - Based on the problem of discrete log over finite field.
  - For a problem a^b = c, ‘a’ and ‘c’ are known, b is required.
  - Can be solved easily using logarithms.
  - For larger number the complexity increases and desired level of security is achieved.

- DSA and RSA are computationally intensive in terms of memory requirement and time.

Elliptical Curves

Basic Properties:

- Equation of an elliptic curve:
  $y^2 = x^3 + ax + b$

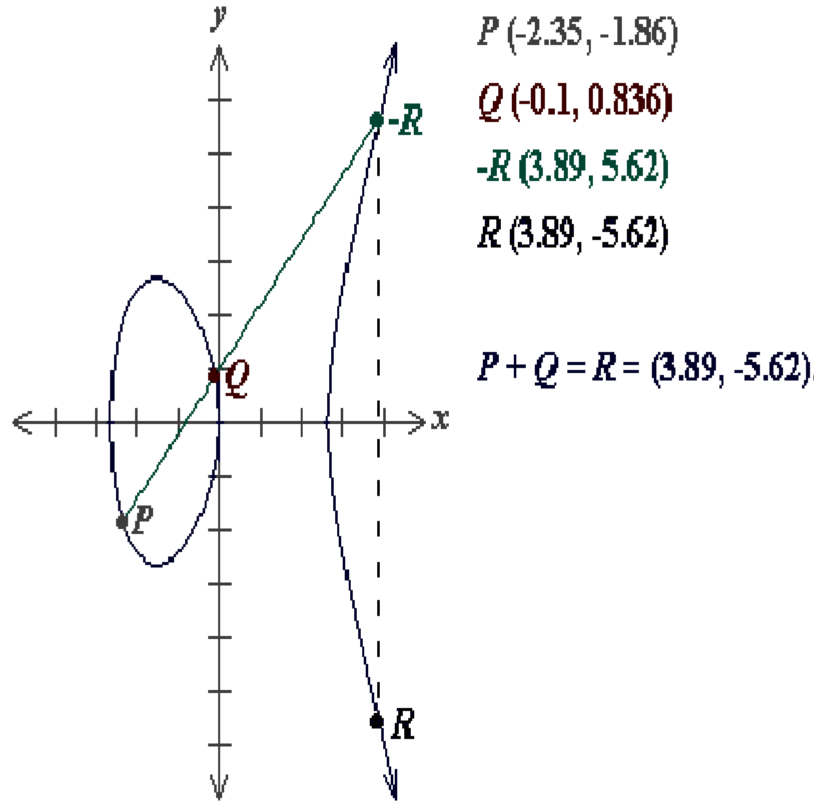
- The equation is defined for no repeated factors.

- Elliptic curve groups are additive groups.

- The addition of any two points on curve is defined geometrically.

- Law of addition:
  P+Q = R.

- The point –R on the curve is reflected on x-axis to point R.

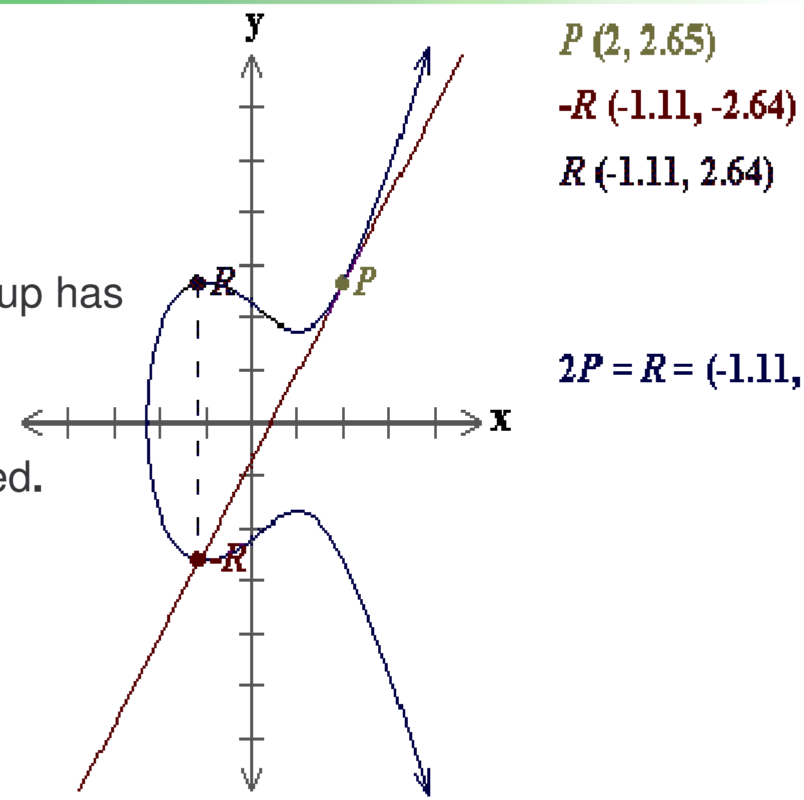


$y^2 = x^3 - 7x$

Source: http://www.certicom.com/index.php?action=ecc_tutorial,ecc_tut_2_1

Elliptical Curves for Cryptography

- Law for doubling a point on elliptic curve
  P+P = 2P = R.
- An essential property for cryptography is that a group has a finite number of points.
- An elliptic curve of underlying field Fp or F_2m is used.
- The modified equation for each underlying field is:
  $y^2 \bmod p = x^3 + ax + b \bmod p$
  $y^2 + xy = x^3 + ax^2 + b$
- Elliptic Curve cryptography is based upon the complexity of discrete log problem.



$y^2 = x^3 - 3x + 5$

Data security and Embedded systems

- Need of data security in Embedded Systems

  - Increase in number of wireless applications
  - Realization of these application on embedded systems platform.

- Current Security Algorithms and Embedded System
  - Time consuming signature generation and authentication.
  - Large key sizes
  - RSA and DSA provide a high level of security, but are expensive in terms of memory.
  - By reducing key size for RSA and DSA, security level is compromised.

- Elliptical Curve DSA (ECDSA)
  - Smaller key sizes without compromising level of security.
  - Quick signature generation and authentication.

ECDSA Implementation

- The elliptic curve discrete log problem:

  Given points P and Q in the group, find a number n such that Pn = Q.

- The private key used is Q.

- Signature generation:

  $r = x\_coord(K = kG) \bmod p$ (For any random number k, with G)
  $s = k$^(-1) ($m + nr$)

- Signature verification:

  - For a user knowing private key Q and verifying signature for message 'm':

    K' = (s^(-1) m) G + (s^(-1) r) Q.
    r' = $x\_coord(K')$

- Accept if r == r'

- Why we obtain smaller key sizes using ECDSA?
  - RSA and DSA complexity increases as the numbers involved increases.
  - The use of finite field and modification in the equation of the curve.

Elliptic-Curve Digital Signature Algorithm (ECDSA)

NIST Guidelines for Public Key Sizes for AES			
ECC key size (bits)	RSA key size (bits)	Key size ratio	AES key size (bits)
163	1,024	1:6	
256	3,072	1:12	128
384	7,680	1:20	192
512	15,360	1:30	256

Supplied by NIST to ANSIX9F1

Table 1

- Advantages of using ECDSA on Embedded systems
  - Signature can be calculated before hand.
  - Smaller key size.
  - Less intensive modular operations.
  - Quick generation of signatures

- Implementation of ECDSA on TI MSP430x33x
  - Acceptable performance at lower cost.
  - Modified underlying field for faster arithmetic operations.
  - Signature generation time is 3.4 sec

- ECDSA on Palm PDAs
  - Signature generation time – 0.9 sec
  - Signature verification time – 2.4 sec
  - 163 bit ECDSA key provides same level of security as 1024 RSA key.

- Conclusion

Performance advantages of ECDSA

- Computationally less intensive than RSA and DSA.
- High security levels in constrained environments.
- Reduction in key size without compromising the data integrity.
- By having smaller key sizes and efficient signature generation ECDSA is extremely suitable for embedded applications.