DESIGN OF INTELLIGENT SPACECRAFT: AN INTERDISCIPLINARY ENGINEERING EDUCATION COURSE

Abstract

This paper discusses a highly interdisciplinary course offered to students during the Spring 2007 semester : Design of Intelligent Spacecraft. The course integrates concepts from mathematics, physics, engineering and computer science for the purpose of educating 4th year undergraduate and introductory masters-level students on the design of intelligent spacecraft. Course content is divided into two pedagogically separate parts :

1. The historical development of physical models, including mathematical models for celestial mechanics and thermodynamics.

2. Application of these models for creating intelligent spacecraft, i.e., applications of these models to pattern recognition, computer vision, and image processing. The first section introduces physical mathematical models which, in the second section of the course, are re-visited to allow for model-based design.

In part (1), a new tact is taken for teaching the historical development of mathematics and physics that shapes the scientific view of the world today. Lectures seek to emphasize the rationale behind scientific thought through the variety of personalities that have defined it best characterized by the phrase : *All science was new at some point*. Specific classical topics include celestial mechanics and thermodynamics which are introduced using excerpts from original works of the scientists that defined and revolutionized our understandings of these fields. Some scientists considered are Aristotle, Tycho, Kepler, Newton, Euler, Bernoulli, Fourier and other scientists relevant to course topics. Where possible, original manuscripts were provided and clarified by reformulating the work in modern terminology and mathematical notation. Historical content is complemented with discussion on contemporary space missions relevant to the discussion topic. For example, historical discussions on the discoveries of Cassini or Galileo includes discussions on the recent Cassini-Huygens mission to Saturn. Further, these discussions include mission spacecraft type, its relevant design considerations and mission objectives. Discussion of mission objectives serve to highlight current boundaries of scientific knowledge and how specific space missions seek to understand topics at these boundaries.

In part (2), students implemented programs relevant to spacecraft design. Programs included physical simulations of celestial mechanics, thermodynamics, and signal processing programs for image manipulation and signal compression. Project topics reinforce topics covered in part (1) of the course. Results for physical simulations are compared against theoretically perfect results for thermodynamic simulations and established gold-standards from NASA's HORIZONS system in

the case of celestial mechanics. Applications of these mathematical models in electrical engineering lead to signal processing projects which motivate subsequent course topics on communication, image processing and image compression.

This paper includes successes, failures and lessons learned in teaching a course with such diverse content and analyzes how well the mixture of history / engineering was received by the students.

Introduction

A new inter-disciplinary course, *Design of Intelligent Spacecraft,* was developed and taught during the 2007 academic year in at the University of North Carolina at Charlotte in the Department of Electrical and Computer Engineering (ECE). Successful spacecraft designed are predicated on the ability for a wide variety of disciplines to contribute sets of compatible instruments and systems which as a whole make up the spacecraft. The course introduces key design issues common to deep-space spacecraft and focuses specifically on terrestrial spacecraft such as the Mars Exploratory Rover (MER) Mission.

The course is not intended to directly address the mechanical issues associated with building of spacecraft. Rather, students are provided conceptual background on the development of astronomy, space exploration, and introductory-level understanding of several prominent design issues that must be solved to design, build, and implement a successful robotic spacecraft.

The overall goal of the course is to introduce junior and senior level undergraduate students and first-year graduate students at the University of North Carolina at Charlotte to design issues relevant in building "intelligent" robotic spacecraft, i.e., spacecraft capable of completing complex mission tasks completely- or semi-automatically.

Since the audience for this course was primarily students within the ECE department, course sections built up areas typically omitted from a Electrical Engineering curriculum. Specifically, the course topics included : the history of astronomy and space exploration, the development of mathematical models in the form of ordinary differential equations (ODEs) and partial differential equations (PDEs), numerical methods to solve ODEs and PDEs, common spacecraft sensors and systems, celestial mechanics and celestial navigation. Material for these topic come from a wide variety of courses in a number of different disciplines including : history, mathematics, physics, engineering and computer science.

As is typically the case for inter-disciplinary courses, there are formidable challenges in presenting this material in an understandable way for the wide variety of students that may enroll for the course. Other challenges include instructing both history and scientific theory simultaneously, and providing a global perspective for the roles of each topics in the overall spacecraft design.

As mentioned earlier, the course is separated into two parts. The pedagogical content of these sections is distinct and are as follows :

1. Topics in astronomy, mathematical modeling, and spacecraft design.

2. Numerical methods for modeling spacecraft design, applications of communications and optical imaging to spacecraft design.

Section (1) comes first since it concentrates on introducing new concepts and theory typically omitted from standard electrical engineering curriculum at both the undergraduate and graduate level. A new tact for teaching classical topics such as mechanics and thermodynamics is taken which uses source materials from the original manuscripts. Although, these manuscripts have limited relevance in terms of modern terminology and notation for these subjects, the experiment here is to evaluate if this new approach stimulates student interest in a subject which, in the typical teaching context, may be dry and uninteresting. The central concept which I seek to convey to students is the realization that *All science was new at some point*.

With the vast attention to detail in many engineering classrooms, this truism is easily forgotten. This is reflected in the often omission of historical context of topics in both engineering texts and lecture halls. Some texts[1] make attempts to provide brief sketches of famous scientists who made fundamental contributions to the discipline, yet these descriptions are often superficial in content and rarely enter discussion in the classroom. The proposed course will use excerpts from the original publications written by the magnates of science and mathematics who have made fundamental contributions to modern science. Since this course will fall into the curriculum of the Electrical and Computer Engineering Department, emphasis will be given to historical figures that are both relevant to the course and relevant to topics from the core departmental curriculum.

Also as part of section (1), historical content is complemented with technical content on the following topics :

- Development of astronomy from ancient times, starting with Aristotle's geocentric model of the universe.

    - Key technical concepts : Observational astronomy : forming images with telescopes, basics of telescope optics, and lenses. Theoretical astronomy : Copernicus's *De revolutionibus orbium coelestium* and the development of the Helocentric theory

- Development of mathematics from ancient times, particularly its role as a tool to model the dynamics of heavenly bodies which came to define the field of celestial dynamics chiefly through Newton's universal theory of gravitation.

    - Key technical concepts : Kepler's Laws for planetary motion, Newton's re-formulation of these laws as a natural consequence of his universal law of gravitation. ODEs that arise from Newtons universal law of gravitation. Thermodynamics and heat dissipation as modeled by a PDE.

Section (2) concentrates on the technical implementation of software algorithms needed to solve the complex problems encountered when building robotic spacecraft. Discussion starts by examining the difficulties that arise when we seek to obtain analytical solutions to the mathematical models developed in section (1). This motivates the use of numerical methods for solving both ODEs and PDEs and the majority of this course section is dedicated to instructing students on the concepts and implementation of these methods. In parallel to lecture, two-three week projects are assigned that require students to implement numerical techniques to address real-world problems. The projects follow:

1. Celestial mechanics : Modeling the Solar System using Runge-Kutta method to solve ordinary differential equations associated with the two-body problem (see §*Appendix B: Celestial Mechanics Project*).

2. Thermodynamics : Modeling Spacecraft Heat Dissipation using Euler's method and the Crank-Nicolson method to solve 1-D and 2-D partial differential equations associated with heat dissipation (see §*Appendix C: Thermodynamics Project*).

3. Signal Processing : Image Formation, Extracting Structure from Images and Image Filtering for Compression via Wavelet Filter Banks (see §*Appendix D: Signal Processing Project*).

Given that undergraduates and graduates are both enrolled in the course, different levels of expertise were required of students based on their enrollment status. For this reason, each project consisted of three *technical levels.* Higher technical levels required more proficiency with the course material. Those students enrolled in the undergraduate section of the course needed to complete only the first technical level within the assigned projects. Students enrolled in the master-level graduate section of the course were required to complete both the first and second technical levels. Students enrolled in the phd-level graduate section of the course were required to complete all three provided technical levels. Students enrolled in the undergraduate and master-level sections were encouraged to attempt higher technical levels for extra credit. This stratification of the project assignments was a very effective in allowing all students to work on the same topics while being required to have different levels of proficiency commensurate to their enrollment status in the course.

Textbook

As is often the case for inter-disciplinary courses, finding an appropriate text is extremely difficult and, in the final analysis, a course textbook was not adopted. As a result, source material for many of the class topics include the entry "Lecture notes" which refer to note hand-compiled from a vast variety of sources during course preparation in the Summer of 2006 (see §*Class Topics*). Many different potential texts were considered primarily focusing on introductory astronomy texts as these course concepts are a major component to the course. Specific texts considered as possible source texts included.[2–4] Other contemporary texts were very valuable as study resources relevant to course topics such as[5] and, of course, a large variety of original scientific manuscripts served as source material, especially for section (1), including.[6–15]

Two texts were read by students to complement lecture topics: *Zero: The Biography of a Dangerous Idea,* by Charles Seife[16] and *The Mathematical Experience,* by Phillip Davis and Reuben Hersh.[17] The first text discusses with the development of numbers, algebra, formal mathematics and calculus accompanied by the historical forces that drove these developments. The second text discusses the mathematics and more advanced mathematical concepts including non-Cantorian set theory and the Reimann hypothesis.

Class Topics / Source Materials / Scoring Rubrics

As mentioned later in this paper (see §*Lessons Learned*) not all of the intended class topics were discussed. In this section, those course topics covered in the course are discussed. The pedagogical content of each topic is These topics differ slightly than those originally laid out in the course syllabus (see §*Appendix A : Initial Course Syllabus*). This section lists each of these topics (in bold) and, along with the topic, key concepts, relevant source material and scoring rubrics are listed.

The history numbers the development of basic mathematics. *Historical concepts:* Rationale behind the development of numbers, the development of algebra, development of different number systems, the history of the Arabic digits. *Source material: Zero: The Biography of a Dangerous Idea,* by Charles Seife and *The Mathematical Experience,* by Phillip Davis and Reuben Hersh. *Rubrics:* Test 1, Test 2, Final Examination, Quizzes and Homeworks.

Outer space and issues in spacecraft construction and system design. *Technical concepts:* What is space? Different types of spacecraft: flyby spacecraft, orbiter spacecraft, atmospheric spacecraft, lander spacecraft, rover spacecraft, penetrator spacecraft, observatory spacecraft, and communications spacecraft. Components to spacecraft: spacecraft cruise module, payload, and propellant. Spacecraft systems: tracking, temperature control, power, communications, science instruments. *Source material:* Lecture notes. *Rubrics:* Test 1, Test 2, Final Examination, Quizzes and Homeworks.

Scientists and their discoveries which contribute to current methods of space exploration. *Historical concepts*: Know the names, chronology, and achievements of scientists that have contributed to the development of course topics such as astronomy, telescope optics, celestial mechanics, and thermodynamics. *Source material*: Lecture notes, original works: *Opticks,* Isaac Newton[6] and *Sidereus Nuncius (Sidereal Messenger),* Galileo Galilei.[11] *Rubrics:* Test 1, Test 2, Final Examination, Quizzes, and Homeworks.

Basic astronomical terminology Astronomical coordinate systems and development of celestial mechanics models. *Technical concepts:* Understand ephemeris variables, how to perform 3-dimensional Euclidean transformations to express the position of points in different coordinate systems, Kepler coordinates, Heliocentric Ecliptic Coordinates, Julian 2000.0 Coordinate System, International Celestial Reference Frame. *Source material:* Lecture notes, original works : *Harmonices Mundi (Harmonies of the World),* Johannes Kepler[10] and *The Principia,* Isaac Newton.[7] *Rubrics:* Final Examination, Project 1.

Mathematical modeling and its application for prediction. Technical concepts: Using ODEs to develop an ephemeris that can predict the location of planets. Using PDEs to predict the temperature distribution within a space capsule exposed to a known constant surrounding temperature, i.e., boundary condition. *Source material:* Lecture notes. *Rubrics:* Project 1, Project 2.

Image formation for astronomy and telescope design, radiometry and simulation of light propagation. *Technical concepts:* The historical development of the telescope, refractive telescopes, Newtonian telescopes, Gregorian telescopes, Schmitt-Cassegrain telescopes, Ritchie-Chretien telescopes, basics of telescopic imaging and very-low power imaging. Digital image formation, sensors for image formation such as CCD and CMOS sensors.

Thermodynamic issues in spacecraft design for deep space exploration. *Technical concepts:* Thermodynamics, heat dissipation, the heat equation, the divergence theorem

Gravitation, Escape Velocity. *Technical concepts:* Newtons Three Laws of Motion, gravity, universal law of gravitation, propulsion, energy to mass ratio

Celestial Mechanics. *Technical concepts:* Kepler's Laws, Ordinary Differential Equations arising from the two-body gravitational problem, NASA's development of an online Ephemeris and how it may be used, extensions to the two-body model and improved models of the dynamics of our solar system through the secular variations of the planetary orbits (VSOP82 and VSOP87).

Celestial Navigation. *Technical concepts:* Stationary points of the two-body model, Libration points, the Interplanetary Transport Network (ITN), Spacecraft propulsion, Hohmann transfer orbit, Gravitational slingshot. *Source materials:* Lecture notes. *Rubrics:* Test 1, Test 2, Final Examination.

Numerical Methods for solving Ordinary Differential Equations. *Technical concepts :* Runge-Kutta method for integrating the ordinary differential equation governing the two-body gravitational problem. *Source materials: Numerical Recipes in C,* Press, Teukolsky, Vetterling and Flannery.[5] *Rubrics:* Project 1, Test 2.

Numerical Methods for solving Partial Differential Equations. *Technical concepts:* Applying finite difference methods for solving PDEs with the specific example of the heat diffusion equation. Explicit methods including forward Euler scheme in both 1 dimension and 2 dimensions. Implicit methods including the Crank-Nicholson scheme in both 1 dimension and 2 dimensions. *Source materials: Time Dependent Problems and Difference Methods*, Gustafsson Kreiss and Oliger,[18] *Numerical Recipes in C,* Press, Teukolsky, Vetterling and Flannery.[5] **Rubrics:** Project 2, Final Examination.

Computer Usage

Students design, simulate, and analyze a variety of projects emphasizing image processing and computer vision as it applies to solving difficult problems in deep space exploration spacecraft design. MATLAB should be accessible as a tool for completing projects.

From a first-day class survey (see §*Teaching the Course*), it was apparent that many of the students had not been exposed to MATLAB programming. Hence, an initial tutorial in-class was provided and several supporting resources were made available through the course website.

Grading Measures

Quantitative measures of student aptitude in the various course concepts were measured using examinations, oral presentations, homeworks, in-class quizzes, and computer programming projects. This section details the method used in applying each measure and presents student performance in each category. Some discussion is provided to detail trends and other notable observations relevant to the measurement and student performance.

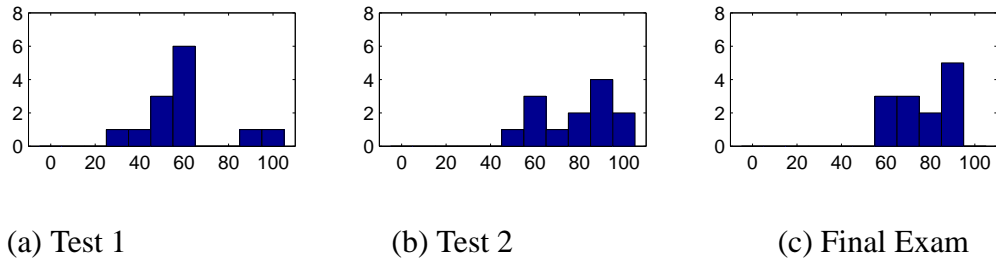(a) Test 1   (b) Test 2   (c) Final Exam

Figure 1: Plots (a-c) above show histograms of the students performance on the three tests administered over the course of the semester. Each test consisted of approximately 50% historical / conceptual questions and 50% technical questions. Students showed improved their performance on tests over the semester.

Examinations

One measure of student proficiency in course topics was provided by midterm and final examinations. These examinations give nearly equal weight to historic and technical course concepts with slightly more emphasis on technical topics.

As one can see from Fig. 1, the students struggled on the first test. It is most likely that this may be attributed to the fact that most students had not previously taken courses with the instructor and that they were not familiar with the examination style of the instructor or accustomed to the degree

Undergraduate Student Scores

(a) Midterm History   (b) Midterm Technical   (c) Final History   (d) Final Technical

Graduate Student Scores

(e) Midterm History   (f) Midterm Technical   (g) Final History   (h) Final Technical
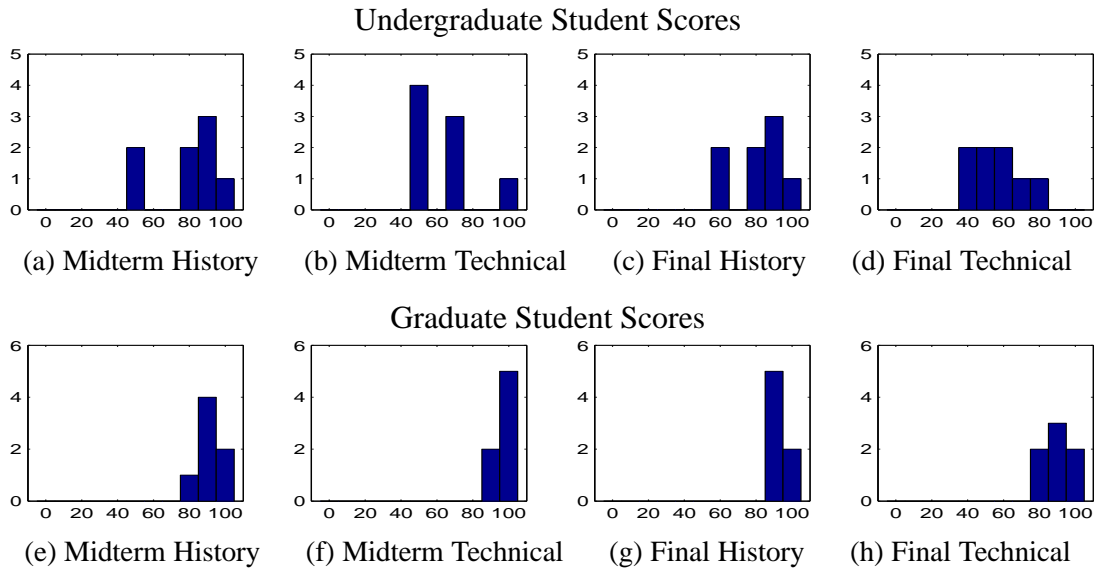
Figure 2: Plots (a-g) above show histograms of the student performance on the historical and technical component of the midterm and final examinations. (a-d) are histograms of undergraduate student grades and (e-h) are histograms of graduate student grades. Students generally scored similarly on the historical aspects of both the midterm and final. However, for the technical half of these examinations, undergraduate students showed improved performance on the final examination while graduate students scored similarly in both historical and technical aspects of these tests.

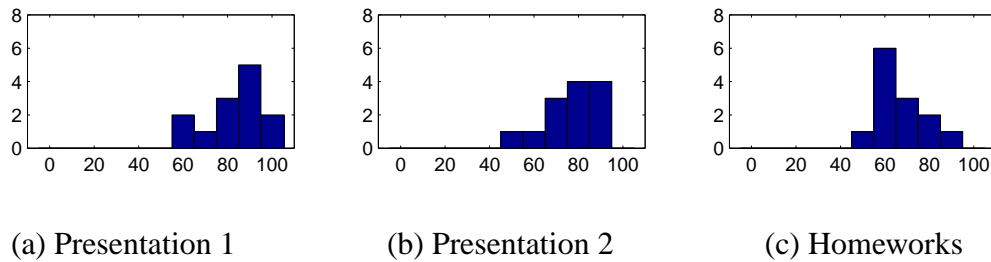(a) Presentation 1   (b) Presentation 2   (c) Homeworks

Figure 3: (a,b,c) Show histogram plots of student grades for two presentations and the overall homework average computed from five homeworks completed during the semester. Students improved their performance on presentations from (a) to (b). The performance on homeworks is somewhat lower reflecting the fact that homeworks typically have a higher degree of difficulty.

of proficiency on the course materials expected by the instructor. This hypothesis agrees with the general trend observed from (a-c) which shows students steadily improving their test scores.

Homeworks / Quizzes

When projects and presentations are not assigned, homework assignments are provided to develop student proficiency in lecture topics and quizzes encourage developing these proficiencies in-sync with their presentation in lecture. During the course of the semester three homeworks were assigned and five quizzes were conducted.

Peer-Reviewed Presentations

Each student provided two presentations and associated written reports during the first half of the course. The first presentation had each student focus on a specific scientist of significance that has made one or more significant contributions to topics of interest for the course. The second and third presentations required each student to read a contemporary research article related to astronomy, or space exploration. *Science* was often used as an appropriate source since it's articles are written to be understandable to a wide readership. Students were asked to present the topic of the research article, the significance of the research and findings in the article and the background information necessary to clarify the significance of the research results.

Student presentations were rated by other students using peer review sheets. The review sheet allowed each student to rate the presentation on a 5 point scale. A rubric was adopted that assigned the overall presentation score to be the average overall score from the student peer-review combined with the instructors rating of the presentation with a 40-60 percent weighting respectively.

From Fig. 3, one can see that students had mixed results in the initial presentation and better results in the second presentation. The overall homework average shown in plot (c) reflects the fact that homeworks had a higher degree of technical difficulty. Proficiency on such homework problems prepare students to improve their performance on measures that more significantly impact their grade such as examinations and projects.

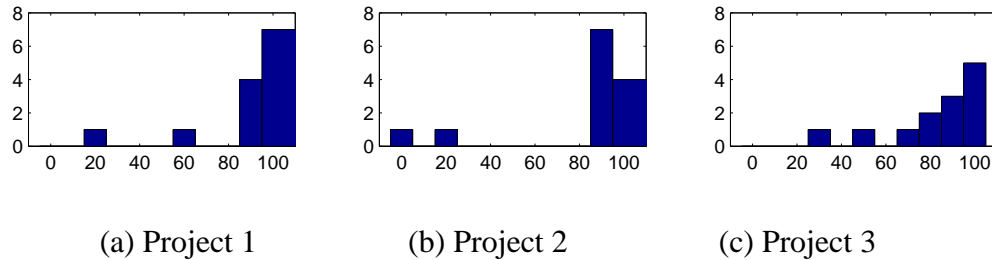(a) Project 1   (b) Project 2   (c) Project 3

Figure 4: (a,b,c) Show histogram plots of student grades for three technical projects. Students performed very well on projects 1 and 2 and some students had difficulty in solving project 3.

Student presentations were provided in electronic format and posted to the course website using a web Content Managent System (CMS). After completion of their presentation, each student provided three questions relevant their presentation. The resulting set of questions were provided to class students as a study guide when reviewing the online presentations and a selected subset of the questions were modified for inclusion in subsequent quizzes and examinations.

Computer Projects

Three computer projects were assigned to students over the course of the semester. Due to the structure of the class, the projects were assigned during the latter half of the class with some overlap into section (1) (the historic component of the course). Project 1 was on celestial mechanics (see §*Appendix B: Celestial Mechanics Project*) and required students to compute the orbit of the planets by solving the system of ODEs that arise from analysis of Newton's two-body equation, i.e., the mutual gravitational forces exerted between the sun and an orbiting planet such as Earth. Students use computer laboratories for the implementation of the design projects outside of class. Project 2 addressed the thermodynamic of heat dissipation (see §*Appendix C: Thermodynamics Project*) and required students to compute both the analytical and numerical solution to the PDE that models heat dissipation known as the heat equation. Students solved the analytical problem using Fourier's technique which is now known as the Fourier series and solved the numerical problem using Euler's forward scheme and the Crank-Nicolson scheme for integrating PDEs. Project 3 addressed image formation, signal processing and image processing concepts. Students were asked to solve problems regarding image formation and write MATLAB programs to automatically extract structure from images (see §*Appendix C: Thermodynamics Project*). Course topics associated with each project are provided in the following list:

1. Celestial Mechanics : Newton's universal law of graviation, Runge-Kutta's method to integrate ODEs, the two-body problem (see §*Appendix B: Celestial Mechanics Project*).

2. Heat Dissipation : Fourier series as analytical solutions to PDEs, numerical schemes for integrating PDEs (see §*Appendix C: Thermodynamics Project*).

3. Signal Processing : Image Formation, Extracting Structure from Images and Image Filtering for Compression via Wavelet Filter Banks (see §*Appendix D: Signal Processing Project*).

Overall Grading Rubric

Grades are determined by performance on exams, homeworks, projects, and in-class presentations. The weight of each item in determining the final grade is as follows: Homeworks 15%, Projects 35%, Presentations 30%, and Exams 20%.

Teaching the Course

Enrollment for the course was open to all junior, senior and graduate students with the pre-requisite of "Permission of Department" which required the consent of the instructor for enrollment. Even with this somewhat awkward requirement for course registration, enrollment for such an elective course was atypically high and included 15 students overall with 8 undergraduate students and 7 graduate students. Attending students were primarily Electrical and Computer Engineering majors with the exception of three students that had majors in Mechanical Engineering, Physics, and Engineering Technology. Classroom content consisted of two 75-minute lectures each week. Much of the course material was compiled from a variety of sources so class attendance, while not figured into the final course grade, was very important for students to obtain the class notes and keep pace with the class topics.

First Day Survey

During the first lecture, a survey was conducted to determine the student demographics and their backgrounds. The survey determined that underlying interests and technical proficiency of class students. Survey data indicated that most students had taken a course in signals and systems (sometimes referred to as linear systems) and about a third of the students had taken digital signal processing. These courses build mathematical modeling skills which play a significant role in the technical aspects of the course. However, it was an experiment to not require one or both of these courses as pre-requisites. About two-thirds of the students had not been exposed to MATLAB programming which is required to complete the three course projects and some of the assigned homework problems. An in-class MATLAB programming tutorial and a host of supporting materials were provided to empower students to complete their programming assignment (see §*Computer Usage*).

Outcomes

According to the initial syllabus (see §*Appendix A : Initial Course Syllabus*), the following competencies should be imparted to the students:

1. An understanding of the mathematical history of astronomy and electrical engineering (assessment by tests/homework).

2. A basic understanding of astronomy including Kepler's laws and their historical development (assessment by projects,tests, and homework)

3. Knowledge of the image formation process and image processing issues related to astronomical images (assessment by projects and homework).

4. Markov chains and Markov Fields and their use for predication and estimation (assessment by projects and homework).

5. Hands-on experience with various computer tools for implementing and testing algorithms for achieving intelligent behaviors (assessment by design project).

While these were the initial intended outcomes of the course, actual implementation of the course fell short of accomplishing all outcomes. Specifically, outcomes (1), (2), (3) and (5) were accomplished in a manner satisfactory to the instructor. Outcome (4) was not covered sufficiently in the scope of the class. Revisions of the syllabus appropriate for future offerings of this course would eliminate (4) and restate the other outcomes to more be more closely aligned with the course content, as instructed. This would include the following new educational outcomes: (1) understanding the development of scientific thought as it relates to our solar system and universe, (2) understanding thermodynamic considerations for spacecraft design, (3) understanding of signal and image processing issues associated with spacecraft design and (4) understanding different approaches to spacecraft design for specific mission objectives.

Student Feedback

On the final day of classes a second survey was conducted to measure student response to the course. We discuss results from survey questions in this section:

1. Did you feel prepared for this course? If not, what courses do you think might be appropriate as a pre-requisite.

   (a) 13 Responses: (10) Yes, (3) No. Those that did not feel prepared each suggested one pre-requisite: astronomy, a MATLAB programming course, and physics/thermodynamics.

2. What was your favorite component (topic/section/project/homework) of this course and why?

   (a) 10 Responses: (6) the three course projects, (2) historical topics, (1) attending the star party, (1) reading *Zero: The Biography of a Dangerous Idea,* by Charles Seife.[16]

3. What was your least favorite component (topic/section/project/homework) of this course and why?

   (a) 11 Responses: (9) reading *The Mathematical Experience,* by Phillip Davis and Reuben Hersh,[17] (1) the image processing project (Project 3), (1) giving oral presentations.

Looking at the responses we can conclude several things about the course:

- *Most students felt prepared for the course.*
  Although three responses indicated a need for other courses the vast majority of students were able to deal with the concepts as presented in class and were able to demonstrate their understanding in the various quantitative measures (see §*Grading Measures*). This is an important outcome as there were initially some concern about the variety of students which were allowed to enroll in the course.

- *Students preferred the assigned project work.*
  Question 2 from the survey indicates that the projects were the most popular aspect of the course. In course projects, students are asked to implement a program to accomplish technical goals. Hence, these are essentially technical aspects of the course which makes one question whether the role of historical contents within the class. The historical aspects of the course are represented explicitly with two students mentioning the historical lectures and one mentioning Seife's book on mathematical history.

- *Students did not like the second text accompanying the course.*
  With nine student responses, it is clear that the inclusion of Davis and Hersh's book, The Mathematical Experience, was not popular with the students. The content of this book is primarily an editorial on the philosophy of mathematics and mathematical modeling. While this is a component of the course content, it is likely that this text will not be included in future teachings of this course.

Lessons Learned

Student feedback and the experience of teaching this course for the first time has provided valuable lessons. The first lesson is that the course tried to include too much new material for a single-semester course, especially considering the lack of a prerequisite for student enrollment and the fact that both undergraduate and graduate students may enroll in the course. These were major contributing factors that made it impossible to satisfactorily cover the final section of the course (see §*Outcomes*). The specific topic of interest omitted was Markov random fields and their application to performing automated tasks from sensed image data. This topic was to extend the discussion on image and signal processing to include how intelligent machines can use these models for decision-making. Example applications of such models include detecting obstacles for terrestrial navigation, i.e., finding rocks or hazardous terrain, or recognizing landmarks in images for self-localization, i.e., determining where in space or on the planet surface the craft is located. Given that the class prerequisites are not changed, it is likely that outcome (4) will be removed from the syllabus in future offerings of this course.

Unfortunately, measured results do not produce conclusive evidence that the inclusion of historical course content stimulates student interest, motivation, or performance in the classroom. The largest impediment to such analysis is the sample size of 15 total students. A more conclusive statement could be made given a larger enrollment. While the student feedback supports the idea that historical context allows students to better understand *why* they are learning a given topic, it is not clear that they learn that topic better or are motivated to be more interested in the topic after being made aware of the history behind the concepts. More data and analysis of that data is necessary to show that this is the case for this interdisciplinary course.

Conclusions

This paper has described a new interdisciplinary course that marries the instruction of at least five different disciplines in a unique way. Course content is intended for upper level undergraduate and introductory graduate students and seeks to expose these students to concepts in astronomy, history, mathematical modeling, physics and engineering. Course content was taken from a number of different contemporary and historical manuscripts which was intended to stimulate student interest. No significant conclusions can be made regarding the impact of these materials on student proficiencies on course outcomes. Yet, positive student feedback on the historical course materials indicate that there may be a link between these materials and achieving course outcomes. Hence, teaching this course provided new insights regarding the inclusion of historical topics in engineering courses. In addition, this paper presents effective rubrics for oral presentations and effective techniques for designing projects that may be completed by both undergraduate and graduate, i.e., students with different enrollment status. The large enrollment for the course (15) relative to that typically observed in other undergraduate/graduate elective courses (4-5) indicates a significant amount of student interest exists in the topic.

The concepts discussed in the class not only develop student skill in the design of spacecraft and spacecraft instrumentation but also are applicable outside this context. In fact, topics proposed for discussion in class are also used in industry for file compression,[19] image segmentation,[20, 21] robotics and numerous other contexts as described in texts.[21] Lecture notes, presentations, laboratories, and visual media generated for the purpose of instructing this course is provided via the university web site (http://www.visionlab.uncc.edu $\implies$ Courses Link). This is an extremely effective way to disseminate the course content to other universities both local and international.

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Appendix A : Initial Course Syllabus

ECGR3090/6090/8090 SYLLABUS

Written by Andrew Willis
Wednesday, 03 January 2007

Syllabus

ECGR3090/ECGR6090/ECGR8090 - Seminar Course : Design of Intelligent Spacecraft (3)

Catalog Data	The course builds upon and synthesizes knowledge from the engineering science, mathematics, and physical sciences stem of core curriculum. The specific topics teach engineering analysis, synthesis, and design, while simultaneously affording an opportunity for the students to investigate an area of specialization. May be repeated for credit.
Lecture Times	Monday and Wednesday 12:30 p.m. - 1:45 p.m.
Lecture Room	Woodward Hall 125
Reference	No required text.
Additional Reference	Discovering the Universe 7th edition, Comins and Kaufmann III, Freeman, 2005. Astronomy 5th Edition, Chaisson and McMillan, Pearson Prentice-Hall, 2007.
Goals	The objective of this course is to provide students with a working knowledge pf algorithms and techniques used in designing intelligent spacecraft, i.e., robotic systems which can autonomously perform complex space-mission tasks.
Prerequisite	Permission of the Department
Class Topic	Numbers and their history Biographies of mathematicians responsible for developing core electrical engineering and signal processing concepts. Basic astronomical terminology Astronomical coordinate systems and development of celestial mechanics models Mathematical modeling and its application for prediction Image formation for astronomy and telescope design Radiometry and Simulation of light propagation Shading and Interreflection Tracking, Structure from motion Issues in spacecraft design for deep space exploration.
Outcomes	The following competencies should be imparted to the students: 1. An understanding of the mathematical history of astronomy and electrical engineering (assessment by homework). 2. A basic understanding of astronomy including Kepler's laws and their historical development (assessment by projects and homework) 3. Knowledge of the image formation process and image processing issues related to astronomical images (assessment by projects and homework). 4. Markov chains and Markov Fields and their use for predication and estimation (assessment by projects and homework). 5. Hands-on experience with various computer tools for implementing and testing algorithms for achieving intelligent behaviors (assessment by design project).
Computer Usage	Students design, simulate, and analyze a variety of projects emphasizing image processing and computer vision as it applies to solving difficult problems in deep space exploration spacecraft design. MATLAB should be accessible as a tool for completing projects.
Laboratory	Students use computer laboratories for the implementation of the design projects outside of class.
Design Content	The design projects for the course vary from semester to semester.
Grading *	Grades are determined by performance on exams, homeworks, projects, and in-class presentations. The weight of each item in determining the final grade is as follows: Homeworks 15% Projects 35% Presentations 30% Exams 20%
Follow-up Courses	N/A
Academic Integrity	Students have the responsibility to know and observe the requirements of the **UNCC Code of Student Academic Integrity (2001-2003 UNCC Catalog, p. 275)** . This code forbids cheating, fabrication or falsification of information, multiple submission of academic work, plagiarism, abuse of academic materials, and complicity in academic dishonesty.
Notes	Semester syllabus will be provided to the students on the first day of class.
Coordinator	A. Willis, Professor of Electrical & Computer Engineering

Appendix B: Celestial Mechanics Project

Project 1 : Celestial Mechanics

This project will familiarize the student with solving Newton's equations for the motion of the heavenly bodies. This document describes 3 technical levels of completion for the project. For ECGR3090 students, only the first technical level needs to be completed. For ECGR6090 students, both the first and second technical levels need to be completed. For ECGR8090 students all technical levels should be completed. Extra credit will be extended to students that complete technical levels beyond their enrolled status in the course. *Please indicate if your project is intended to include extra credit work.*

Technical Level 1

Using MATLAB, implement a model of the Earth's orbit around the sun using as initial data values for position and velocity taken from the textfile *earthmoon.txt* obtained from NASA's Jet Propulsion Laboratory (JPL) Horizons project. The model should plot the orbit of the Earth using only the 2-dimensional (x,y) coordinates of the Earth. The differential equations of motion should be solved using your implementation of the 4th order accurate Runge-Kutta scheme.

To turn in :

1. Turn in a plot of the Earth's orbit over a 5 year interval using your solution and using the analytic solution.

2. Turn in a plot showing the error between the Earth's orbit as represented by Kepler's analytic solution and the solution computed using Runge-Kutta numerical integration. To do so, find the minimum distance between each computed Kepler orbital position and the closest orbital position of the Earth using the Runge-Kutta method. Plot the magnitude of this difference as a function of the position index.

3. Turn in a plot of the error magnitudes between your computed positions for the Earth and the positions computed by JPL using the Horizons program for the time interval January 1, 2000 to January 1, 2005. Here, your x-axis should be the point index for each of the JPL data points and the y-axis should be the minimum distance between a specific JPL data point and the points computed from Runge-Kutta integration.

4. Turn in your MATLAB code.

5. Turn in a discussion of what you observe from the plots generated in (1) and (2).

Technical Level 2

Extend the results from technical level 1 to 3-dimensional space and add Jupiter, Mars, Venus and Saturn. Initial positions and velocities of these planets are available from textfiles that bear the planet name, e.g., *venus.txt*.

To turn in :

1. Turn in a single plot showing the orbits of all the planets over a 5 year interval using your solution.

2. Turn in a plot that compares your solution to Kepler's analytic solution for each of the planet by plotting both solutions and taking the difference of the length of the position vectors. (Similar to that from Technical Level 1, but the vectors are now 3D).

3. Turn in a plot which shows, as different traces, the error magnitudes between your computed positions for each of the planets and the positions computed by JPL using the Horizons program for the time interval January 1, 2000 to January 1, 2005. The plot should also have a legend. (Similar to that from Technical Level 1, but the vectors are now 3D).

4. Turn in your MATLAB code.

5. Turn in a discussion of what you observe from the plots generated in (1) and (2).

Technical Level 3

Extend the results from technical level 2 and add Mercury to the 3-dimensional solar system model. Read, understand and incorporate corrections associated with relativistic effects as described in Aldo Vitagliano's paper *"Numerical integration for the real time production of fundamental Ephemerides over a wide time span"* (ask for additional handout regarding this problem).

To turn in :

1. Turn in a single plot showing the orbits of all the planets over a 5 year interval using your solution and using the analytic solution.

2. Turn in a plot that compares your solution to Kepler's analytic solution for each of the planet by plotting both solutions and taking the difference of the length of the position vectors. (Similar to that from Technical Level 2).

3. Turn in a plot of the error magnitudes between your computed positions for each of the planets and the positions computed by JPL using the Horizons program for the time interval January 1, 2000 to January 1, 2005. (Similar to that from Technical Level 2).

4. Turn in your MATLAB code.

5. Turn in a discussion of what you observe from the plots generated in (1) and (2).

6. Turn in a discussion on the significance of relativity in implementing your solar system model and how significant this modification was for the planet mercury.

Appendix C: Thermodynamics Project

Project 2 : Heat Diffusion

This project will familiarize the student with thermodynamic modeling of heat flow through Fourier series and using finite difference schemes to solve the partial differential equation describing this process. This document describes 2 technical levels of completion for the project. For ECGR3090 students, only the first technical level needs to be completed. For ECGR6090 students, both the first and second technical levels need to be completed. For ECGR8090 students all technical levels should be completed. Extra credit will be extended to students that complete technical levels beyond their enrolled status in the course. *Please indicate if your project is intended to include extra credit work.*

Using MATLAB, implement a model of heat diffusion by solving the heat equation using finite differencing techniques as discussed in class. We will use a metal bar with heat conductivity constant $\alpha = 1$ and length $L = 10$ as our experimental object. Take the initial heat profile for the bar is given in (1).

$$u(0,x) = \begin{cases} 1 & 4 \leq x \leq 6 \\ 0 & x < 4, \ x > 6 \end{cases} \tag{1}$$

The boundary conditions for our project will include holding the temperature to a constant 0 degrees Celcius at the boundaries, i.e., $u(t,0) = 0 = u(t,L)$ where L denotes the x position at the end of the metal bar, i.e., $L = 10$.

We will use the heat equation (2) to model the heat diffusion process.

$$u_t = \alpha u_{xx} \tag{2}$$

For our situation, we will compute two solutions : (1) the analytic solution obtained by solving the partial differential equation (PDE) and (2) the numerical solution computed using a finite differencing scheme.

Technical Level 1

For technical level 1, we will integrate the 1-dimensional heat equation for the provided initial conditions using Euler's method to integrate the PDE.

To turn in :

1. **On a single piece of paper**, using MATLAB's subplot function, turn in plots of the numerically computed temperature distribution $u(t_0,x)$ for times $t_0 = \{0,1,2,3,4,5\}$ secs. For each plot, overlay the approximate analytic solution using $N = 50$ sinusoidal functions at the corresponding time instant. The equation for these sinusoids is available in the handout on thermodynamics.

2. Turn in a plot of the error magnitudes between your computed temperatures and the analytic temperatures for each of the 6 plots.

3. Re-run your experiment as in part (1) but change the heat conductivity to $\alpha = 2$ and the number of approximating sinusoids to $N = 10$. Plot all of these on a single piece of paper. Note this may will require your timestep k to be smaller to be stable.

4. Turn in a plot of the error magnitudes between your computed temperatures and the analytic temperatures for each of the 6 plots.

5. Turn in your MATLAB code.

6. Turn in a discussion of what you observe from the plots generated in (1-4). Explain the effects of the changes made between (1) and (3).

Technical Level 2

Your first task will be to implement technical level 1 in it's entirety. You will then solve these equations using the more accurate Crank-Nicolson method and finally extend these results from 1-dimensional space to 2-dimensional space. You will use the more accurate Crank-Nicolson method to compute your solutions in 1D and you will use Euler's method to compute your solutions in 2D. As before, we will use a square slab of metal with heat conductivity constant $\alpha = 1$ and lengths $L_x = L_y = 10$ in the x and y directions as our experimental object. Take the initial heat profile for the bar is given in (1).

$$u(0,x,y) = \begin{cases} 1 & 4 \leq x \leq 6, 4 \leq y \leq 6 \\ 0 & elsewhere \end{cases} \tag{3}$$

We will use the 2D heat equation (2) to model the heat diffusion process.

$$u_t = \alpha(u_{xx} + u_{yy}) \tag{4}$$

The boundary conditions for our project will include holding the temperature to a constant 0 degrees Celcius at the boundaries, i.e., $u(t,0,y) = u(t,L_x,y) = u(t,x,0) = u(t,x,L_y) = 0$ where L_x denotes the x position at the end of the metal slab and L_y denotes the y position at the end of the metal slab.

To turn in :

1. Implement technical level 1 and turn in all material required in the technical level 1 assignment.

2. Implement the Crank-Nicolson method to numerically integrate the 1D heat equation and repeat technical level 1. Explain any differences you observe in the error magnitudes.

3. Implement the Euler method to to solve the 2D heat equation with the initial conditions (3). On a single sheet of paper, provide *mesh()* plots of the solutions for times $t_0 = \{0,1,2,3,4,5\}$.

4. Turn in your MATLAB code.

5. Turn in a discussion of what you observe from the plots generated in (2) and (3).

Technical Level 3

Your first task will be to implement technical levels 1 and 2 in their entirety. You will then solve the 2D heat equation using the more accurate Crank-Nicolson method.

To turn in :

1. Implement technical level 1 and turn in all material required in the technical level 1 assignment.

2. Implement technical level 2 and turn in all material required in the technical level 2 assignment.

3. Implement the Crank-Nicolson method to solve the 2D heat equation (2) with the initial conditions (3).

4. Compare your results to those in part (3) from technical level 2. On a single sheet of paper, provide *mesh()* plots of the solutions for times $t_0 = \{0, 1, 2, 3, 4, 5\}$.

5. Turn in your MATLAB code.

Possible Extra Credit Extensions :

Allow for different rate propagation in the x and y directions by solving the non-homogeneous heat equation (5). Use the same boundary and initial conditions as specified in technical level 2 and allow $\alpha_x = 1$ and $\alpha_y = 3$.

$$u_t = \alpha_x u_{xx} + \alpha_y u_{yy} \tag{5}$$

Appendix D: Signal Processing Project

Project 3 : Edge Detection and Image Filters in the Frequency Domain

Written Problems

1. A CCD camera chip of dimensions 7 x 7 mm. and having 1024 x 1024 elements, is focused on a square, flat area, located 0.5 m away. How many line pairs per mm will this camera be able to resolve? The camera is equipped with a 35-mm lens. (Hint: Model the imaging process as discussed in class, with the focal length of the camera lens substituting for the focal length of the eye).

2. A common measure of transmission for digital data is the baud rate, defined as the number of bits transmitted per second. Generally, transmission is accomplished in packets consisting of a start bit, a byte (8 bits) of information, and a stop bit. Using these facets, answer the following: (a) How many minutes would it take to transmit a 1024 x 1024 image with 256 gray levels using a 56K baud modem. (b) What would the time be at 750K baud, a representative speed of a phone DSL (digital subscriber line) connection?

MATLAB Task 1 Edge Detection

Write a MATLAB function [absgrad,xgrad,ygrad,thetagrad] = edgeDetect(I,h_avg,ftype), i.e., a 2D function which computes edges within the image I. As discussed in class, computation of image edges is a 3-part process :

1. Smooth the image : Convolves the filter, h_avg, with the image I. *You may use your own convolve2D() function or MATLAB's built-in conv2() function to accomplish this task. Note : if you use MATLAB's built-in function, you will need to crop the resulting image Inew=conv2(I,h_avg) as it will be larger than the dimensions of the original image I.*

2. Compute the image gradient : This is typically accomplished by convolving the smoothed image with a pre-defined filter. The specific filter coefficients will depend upon the value of the *string* ftype. This string will have 3 possible values :

    (a) 'canny' - The Canny edge detector. The coefficients for this filter are simply the coefficients of the filter for the discrete difference operator, i.e., $\frac{\partial I}{\partial x} = f(x,y) - f(x-1,y)$ and $\frac{\partial I}{\partial y} = f(x,y) - f(x,y-1)$.

    (b) 'sobel' - The Sobel edge detector. Given in the handout.

    (c) 'roberts' - The Roberts edge detector. Given in the handout.
    The values for $\frac{\partial I}{\partial x}$ and $\frac{\partial I}{\partial y}$ for each (x,y) pixel location should be placed in variables *xgrad* and *ygrad* respectively.

3. Compute the edge strength at each (x, y) pixel location as shown in equation (6) and place the result in variable *absgrad*.

$$e_s(x, y) = \sqrt{\frac{\partial I(x, y)}{\partial x}^2 + \frac{\partial I(x, y)}{\partial y}^2} \tag{6}$$

Compute the edge orientation at each (x, y) pixel location as shown in equation () and place the result in variable *thetagrad*.

$$e_o(x, y) = \tan^{-1}\left(\frac{\frac{\partial I(x, y)}{\partial y}}{\frac{\partial I(x, y)}{\partial x}}\right)$$

Plot three images in a 3x2 array using the *subplot()* command. The index of the images and the function parameter values for each of the results are indicated below :

1. The original image, I=hw3_image0.jpg.

2. abs(xgrad) from edgeDetect(I, h_{avg}- the 8x8 averaging filter,'canny').

3. abs(ygrad) from edgeDetect(I, h_{avg}- the 8x8 averaging filter,*'canny'*).

4. absgrad from edgeDetect(I, h_{avg}- the 8x8 averaging filter,'canny').

5. absgrad from edgeDetect(I, h_{avg}- the 8x8 averaging filter,'roberts').

6. absgrad from edgeDetect(I, h_{avg}- the 8x8 averaging filter,'sobel').

MATLAB Task 2 Spatial Filters and Frequency Domain Analysis

Plot the magnitude response (in dB) of the 1-dimensional discrete fourier transform (DFT) of the 1-dimensional version of the following filters discussed in class :

1. the Laplacian filter : $h[n] = [1, -2, 1]$

2. the Averaging filter that averages 10 samples : $h[n] = [\frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}]$

Make sure that the filters have *exactly* a width of 301 samples (padding zeros where necessary).

Next multiply (modulate) the filters by $(-1)^n$. For example the Laplacian filter becomes $h[n] = [1, 2, 1]$.

Plot the magnitude responses for these four Fourier transforms in a 2x2 array of plots. Note the cutoff frequencies for each filter, i.e., points where the magnitude plot intersects the line at -3 dB.

```
x_fft = fft(x);
plot(abs(x_fft))
```

Note that the effect of multiplying by $(-1)^n$ modulates the signal and moves the index of the DC component to $\frac{N}{2}$ for a DFT containing N points. Hence, we may lowpass filter a 1-d signal $x[n]$ using the following steps :

1. Modulate the signal by computing $y[n] = (-1)^n x[n]$

2. Compute the $Y[k] = fft(y[n])$

3. If $y[n]$ has length N, low pass filter $Y[k]$ by defining

$$H[k] = \begin{cases} 0 & 0 \leq k \leq \frac{N}{2} - k_0, \frac{N}{2} + k_0 \leq k \leq N \\ 1 & \frac{N}{2} - k_0 \leq k \leq \frac{N}{2} + k_0 \end{cases}$$

4. Compute $G[k] = Y[k] * H[k]$

5. Compute $g[n] = ifft(G[k])$

6. Compute $f[n] = (-1)^n g[n]$.

7. Plot the filtered signal $f[n]$, it is a low-pass filtered version of $x[n]$

Likewise, we may filter a 2-dimensional signal. By generalizing this method :

1. Modulate the signal by computing $y[i, j] = (-1)^{i+j} x[i, j]$

2. Compute $Y[m, n] = fft2(y[i, j])$

3. If $y[i, j]$ has dimensions M, N define the low pass filter $H[m, n]$ assuming a value k_0

$$H[m, n] = \begin{cases} 1 & \frac{M}{2} - k_0 \leq m \leq \frac{M}{2} + k_0, \frac{N}{2} - k_0 \leq n \leq \frac{N}{2} + k_0 \\ 0 & elsewhere \end{cases}$$

4. Compute $g[i, j] = ifft(Y[m, n] * H[m, n])$

5. Compute $f[i, j] = (-1)^{i+j} g[i, j]$

6. Plot the filtered signal $f[i, j]$, it is a low-pass filtered version of $x[i, j]$

The variable k_0 determines the frequency cutoff value of the filter. In 1D $k_0 \in [0, ..., \frac{N}{2}]$ with smaller values of k_0 passing lower and lower frequencies. In 2D $k_0 \in [0, ..., min(\frac{M}{2}, \frac{N}{2})]$ where smaller values of k_0 also passes lower frequencies.

Take the audio file *mary.wav* and read it into MATLAB using *wavread()* command. Then filter the signal using the 1-dimensional low-pass filter described above.

1. Plot $x[n]$ the input signal for times $t = [0.5, 0.6]$ seconds.

2. Plot $|X[k]|$ the magnitude response for the signal $x[n]$.

3. Plot $|X_{mod}[k]|$ the modulated magnitude response for the signal $x[n]$.

4. Plot $|H[k]|$ the magnitude response for the filter assuming $k_0 = 100$.

5. Plot $|X[k]H[k]|$ the magnitude response for the filtered signal.

6. Plot $g[n]$ the filtered time domain signal.

Take the image file *lena.gif* and read it into MATLAB using the *imread()* command. Then filter the image using the 2-dimensional low-pass filter described above.

1. Plot $I[i, j]$ the input image.

2. Plot $|X[i, j]|$ the magnitude response for the image $I[i, j]$.

3. Plot $|X_{mod}[i, j]|$ the modulated magnitude response for the image $I[i, j]$.

4. Plot $|H[i, j]|$ the magnitude response for the 2D filter assuming $k_0 = 80$.

5. Plot $|X[i, j]H[i, j]|$ the magnitude response for the filtered signal.

6. Plot $g[i, j]$ the filtered image.

Completing your code and annotating your Results

Complete your MATLAB assignment by annotating your displayed results and commenting your written MATLAB functions.

To Turn in :

1. Solutions to written problems.

2. Printouts of the MATLAB files *edgeDetect.m.*

3. Printouts of your MATLAB filter programs for task 2.

4. Pages showing the annotated image plots for the requested results from tasks 1 and 2.

5. Two short paragraphs describing what you have learned / observed from completing each of the tasks.