hw solutions
section 3.2

2. $f(x) = 3x^2(x-1)$

way 1: Product rule

$$f'(x) = 6x(x-1) + 3x^2 = 9x^2 - 6x$$

way 2: simplification

$$f(x) = 3x^3 - 3x^2$$
$$f'(x) = 9x^2 - 6x$$

17. $f(x) = \dfrac{x-1}{2x+1}$

$$f'(x) = \frac{(1)(2x+1) - (x-1)(2)}{(2x+1)^2}$$
$$= \frac{2x+1-2x+2}{(2x+1)^2} = \frac{3}{(2x+1)^2}$$

32. $f(1) = 2$, $f'(1) = -1$, $g(1) = -2$, $g'(1) = 3$

find the value of $h'(1)$.

$$h(x) = (x^2+1)g(x)$$
$$h'(x) = 2x\,g(x) + (x^2+1)g'(x)$$
$$h'(1) = 2(1)\cdot g(1) + (1^2+1)\cdot g'(1)$$
$$= 2(-2) + 2(3)$$
$$= -4 + 6 = 2$$

36. find $f'(2)$. $f(x) = \dfrac{2x+1}{2x-1}$

$$f'(x) = \frac{2(2x-1) - (2x+1)(2)}{(2x-1)^2}$$
$$f'(2) = \frac{2\cdot 3 - 5\cdot 2}{3^2} = \frac{-4}{9}$$

45. Find the tangent to $f(x) = (x^3+1)(3x^2-4x+2)$ at $(1,2)$

$$f'(x) = 3x^2(3x^2-4x+2) + (x^3+1)(6x-4)$$
$$m = f'(1) = 3(3-4+2) + (2)(6-4)$$
$$= 3(1) + 2(2) = 7$$

$$y = mx + b$$
$$2 = 7(1) + b, \quad b = -5$$
$$y = 7x - 5$$

61. True/False: $\dfrac{d}{dx}[fg] = f'g'$

false, need product rule

ex: $\dfrac{d}{dx}[x^2\cdot x] \neq 2x\cdot 1 = 2x$

$= \dfrac{d}{dx}[x^3] = 3x^2$

8. $f(x) = (x^3 - 12x)(3x^2 + 2x)$

way 1:

$$f'(x) = (3x^2-12)(3x^2+2x) + (x^3-12x)(6x+2)$$
$$= 15x^4 + 8x^3 - 108x^2 - 48x$$

way 2:

$$f(x) = 3x^5 + 2x^4 - 36x^3 - 24x^2$$
$$f'(x) = 15x^4 + 8x^3 - 108x^2 - 48x$$

20. $f(u) = \dfrac{u}{u^2+1}$

$$f'(u) = \frac{(1)(u^2+1) - u(2u)}{(u^2+1)^2}$$
$$= \frac{u^2+1-2u^2}{(u^2+1)^2} = \frac{1-u^2}{(u^2+1)^2}$$

33. $f(1) = 2$, $f'(1) = -1$, $g(1) = -2$, $g'(1) = 3$

find the value of $h'(1)$.

$$h(x) = \frac{x f(x)}{x + g(x)}$$
$$h'(x) = \frac{[1\cdot f(x) + x f'(x)](x+g(x)) - x f(x)[1+g'(x)]}{(x+g(x))^2}$$
$$h'(1) = \frac{[f(1)+f'(1)](1+g(1)) - f(1)[1+g'(x)]}{(1+g(1))^2}$$
$$= \frac{[2-1](1-2) - 2[1+3]}{(1-2)^2} = -9$$

43. $g(x) = x^2 f(x)$ & $f(2) = 3$, $f'(2) = -1$. find $g'(2)$.

$$g'(x) = 2x f(x) + x^2 f'(x)$$
$$g'(2) = 2\cdot 2\cdot f(2) + 2^2\cdot f'(2)$$
$$= 4\cdot(3) + 4\cdot(-1) = 8$$

54. dosage: $D(t) = \dfrac{500t}{t+12}$

find $D'(t)$ and $D'(6)$, $D'(10)$

$$D'(t) = \frac{500(t+12) - 500t(1)}{(t+12)^2} = \frac{6000}{(t+12)^2}$$
$$D'(6) = \frac{500}{27} = 18.5185$$
$$D'(10) = \frac{6000}{(10+12)^2} = \frac{1500}{121} = 12.39669$$

63. True/false: $\dfrac{d}{dx}\left[\dfrac{f(x)}{x^2}\right] = \dfrac{f'(x)}{2x}$

false, need quotient rule

ex: $\dfrac{d}{dx}\left[\dfrac{x^3}{x^2}\right] \neq \dfrac{3x^2}{2x} = \dfrac{3}{2}x$

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$\dfrac{d}{dx}[x] = 1$