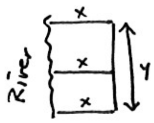
hw solutions
section 4.5

4. 3000 yd fencing
find the dimensions
find the area
(maximizing)



$$3x + y = 3000 \implies y = 3000 - 3x$$

$$A(x) = xy = x(3000-3x) = 3000x - 3x^2$$

$$A'(x) = 3000 - 6x = 0$$

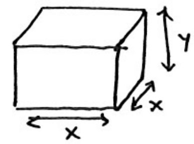
$$x = \frac{3000}{6} = 500$$

$$y = 3000 - 3(500) = 1500$$

dimensions: $500' \times 1500'$

Area: 750,000

9. find the dimensions of a box with square base, volume of 128 in^3 & is constructed using the least material posible.



Volume: $x^2y = 128$

So, $y = 128/x^2$

$$S.A(x) = 2\cdot x^2 + 4xy$$

$$= 2x^2 + 4x\left(\frac{128}{x^2}\right) = 2x^2 + \frac{512}{x}$$

$$SA'(x) = 4x - \frac{512}{x^2} = 0$$

$$4x^3 = 512$$

$$x^3 = 128 \implies x = \sqrt[3]{128} = 4\sqrt[3]{2}$$

$$y = \frac{128}{(128)^{2/3}} = 128^{1/3} = 4\sqrt[3]{2}$$

dimensions: $4\sqrt[3]{2} \times 4\sqrt[3]{2} \times 4\sqrt[3]{2}$

26. see diagram p321

Volume: $16\pi = \pi(r+\frac{1}{2})^2(h+1)$

find r & h when V(r) is maximized

$$\frac{16}{(r+1/2)^2} - 1 = h$$

$$V(r) = \pi r^2 h = \pi r^2\left[\frac{16}{(r+1/2)^2} - 1\right]$$

$$V'(r) = 2r\pi\left[\frac{16}{(r+1/2)^2} - 1\right] + \pi r^2\left[\frac{-32}{(r+1/2)^3}\right] = 0$$

$$\pi r\left[\frac{32}{(r+1/2)^2} - 2 - \frac{32r}{(r+1/2)^3}\right] = 0$$

$$\frac{32}{(r+1/2)^2} - 2 - \frac{32r}{(r+1/2)^3} = 0$$

$$32(r+1/2) = 2(r+1/2)^3 + 32r$$

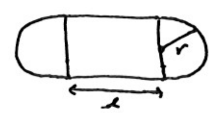
$$16 = 2r^3 + 3r^2 + 1.5r + .25$$

$$2r^3 + 3r^2 + 1.5r - 15.75 = 0$$

$r = 1.5$ (by calculator)

$$h = \frac{16}{2^2} - 1 = 3$$

29. length of track = $1760' = 1/3$ mi
find ℓ & r to maximize the rectangular region.
What is that area?



$$1760 = 2\ell + 2\pi r$$

So, $\ell = 880 - \pi r$

$$\text{Area} = r\ell = r(880 - \pi r) = 880r - \pi r^2$$

$$A' = 880 - 2\pi r = 0$$

$$r = \frac{880}{2\pi} = \frac{440}{\pi} \approx 140.056 \text{ ft}$$

$$\ell = 880 - \pi\left(\frac{440}{\pi}\right) = 440 \text{ ft}$$

$$\text{Area} = \cancel{\left(\tfrac{440}{\pi}\right)440 = \tfrac{193600}{\pi} \approx 61,624.19397 \text{ ft}^2}$$

$$= 2r\ell + \pi r^2$$

$$= 2\cdot\frac{440}{\pi}\cdot 440 + \pi\left(\frac{440}{\pi}\right)^2 = \frac{387200}{\pi} + \frac{193600}{\pi}$$

$$= \frac{580800}{\pi} = 184,874.3819$$

32. total # containers: 1,000,000
setup cost per run: \$500
manufacturing cost: \$.50 / container
storage cost: \$.40 / container
how many containers should be produced per run to minimize cost?

x = # containers per run

avg # containers in storage: $\frac{x+0}{2} = \frac{x}{2}$

storage cost per yr: $(.40)(\frac{x}{2}) = .2x$

# of runs: $\frac{1,000,000}{x}$

manufacturing cost (all containers):

$(.50)(1,000,000) = 500,000$

cost per run: $(500)\left(\frac{1,000,000}{x}\right) = \frac{500,000,000}{x}$

total cost: $C(x) = \frac{500,000,000}{x} + 500,000 + .2x$

$$C'(x) = -\frac{500,000,000}{x^2} + .2$$

$$C''(x) = \frac{1,000,000,000}{x^3}$$

$$0 = \frac{-500,000,000}{x^2} + .2$$

$$.2x^2 = 500,000,000$$

$$x = \pm 50,000$$

Ans: 50,000 containers should be produced per run.