hw solutions
section 5.4

6. $f(x) = 2e^x - x^2$

$f'(x) = 2e^x - 2x$

11. $f(x) = 3(e^x + e^{-x})$

$f(x) = 3e^x + 3e^{-x}$

$f'(x) = 3e^x - 3e^{-x}$

23. $f(x) = e^{\sqrt{x}}$

$f'(x) = e^{\sqrt{x}} \cdot \frac{d}{dx}\sqrt{x}$

$= e^{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2}$

$= \dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

29. $f(x) = e^{-4x} + 2e^{3x}$

$f'(x) = -4e^{-4x} + 6e^{3x}$

$f''(x) = 16e^{-4x} + 18e^{3x}$

38. Determine concavity for $f(x) = xe^x$

$f'(x) = 1 \cdot e^x + xe^x$

$f''(x) = e^x + 1 \cdot e^x + xe^x$

$0 = 2e^x + xe^x$

$0 = e^x(2+x)$

but $e^x \neq 0$

so $2 + x = 0 \Rightarrow x = -2$

$\longleftarrow \; - - - \; \underset{-2}{|} \; + + \; \longrightarrow$

cu: $(-2, \infty)$

CD: $(-\infty, -2)$

10. $f(x) = \dfrac{x}{e^x}$

way 1:

$f'(x) = \dfrac{(1)e^x - xe^x}{(e^x)^2} = \dfrac{e^x(1-x)}{(e^x)^2} = \dfrac{1-x}{e^x}$

way 2:

$f(x) = xe^{-x}$

$f'(x) = e^{-x} + x(-e^{-x}) = e^{-x} - xe^{-x} = (1-x)e^{-x}$

25. $f(x) = (x-1)e^{3x+2}$

$f'(x) = (1)e^{3x+2} + (x-1)(3e^{3x+2})$

$= e^{3x+2} + 3xe^{3x+2} - 3e^{3x+2}$

$= 3xe^{3x+2} - 2e^{3x+2}$

$= (3x-2)e^{3x+2}$

33. $y = e^{2x-3}$ find the tangent line at $(\frac{3}{2}, 1)$

$y' = 2e^{2x-3}$

$m = y'\big|_{x=3/2} = 2e^{2(3/2)-3} = 2e^0 = 2$

$y = mx + b$

$1 = 2(3/2) + b$, $b = -2$

$y = 2x - 2$

44. find the abs extrema of $h(x) = e^{x^2-4}$ on $[-2, 2]$

$h'(x) = 2xe^{x^2-4} = 0 \Rightarrow x = 0$

$h(-2) = e^{(-2)^2-4} = e^0 = 1$

$h(0) = e^{0-4} = e^{-4}$

$h(2) = e^{(2)^2-4} = e^0 = 1$

abs minima: $y = e^{-4}$ (at $x=0$)

abs maxima: $y = 1$

59. $N(t) = 5.3e^{.095t^2 - .85t}$, $0 \le t \le 4$

$t = 0 \rightarrow 1959$

(a) Show $N(t)$ decreases over $[0,4]$

$N'(t) = 5.3(.19t - .85)e^{.095t^2 - .85t}$

$= \underbrace{(5.3)(.19)}(t - 4.4737)\underbrace{e^{.095t^2-.85t}}$

always positive

for any t in $[0,4]$, $t - 4.4737 < 0$

so $N'(t) < 0$ on $[0,4]$

(b) how fast was polio decreasing at 1959? $N'(0) = -4.505$

how fast was polio decreasing at ~~1963~~? $\cancel{N'(4) = -.072785}$
1962

$N'(3) = -.27246$