

hw solutions
section 5.5

1. $f(x) = 5 \ln x$

$$f'(x) = 5 \cdot \frac{1}{x} = \frac{5}{x}$$

5. $f(x) = \ln x^8$

way 1:
 $f'(x) = \frac{8x^7}{x^8} = \frac{8}{x}$

way 2:
 $f(x) = 8 \ln x$
 $f'(x) = \frac{8}{x}$

27. $f(x) = e^x \ln x$

$$\begin{aligned} f'(x) &= e^x \ln x + e^x \cdot \frac{1}{x} \\ &= e^x [\ln x + \frac{1}{x}] \end{aligned}$$

39. $f(x) = x^2 \ln x$

$$\begin{aligned} f'(x) &= 2x \ln x + x \\ &= x(1+2 \ln x) \end{aligned}$$

$$\begin{aligned} f''(x) &= 1+2 \ln x + x(\frac{2}{x}) \\ &= 3+2 \ln x \end{aligned}$$

50. $y = x^{\ln x}$

$$\begin{aligned} \ln y &= \ln x^{\ln x} \\ \ln y &= (\ln x)^2 \\ \frac{y'}{y} &= 2 \ln x \cdot \frac{1}{x} \\ y' &= \frac{2 \ln x}{x} \cdot x^{\ln x} \\ &= 2 \ln x \cdot x^{\ln x-1} \end{aligned}$$

54. find tangent line of $y = \ln x^2$ at $(2, \ln 4)$

$$y' = 2/x$$

$$m = 2/2 = 1$$

$$y = mx+b \Rightarrow \ln 4 = 1(2)+b, b = \ln 4 - 2$$

$$y = x + \ln 4 - 2$$

65. $f(x) = 7.2956 \ln(.0645012x^{.95} + 1)$

find $f'(100)$ & $f'(500)$

$$f'(x) = \frac{-.447046207x^{-.05}}{.0645012x^{.95} + 1}$$

2. $f(x) = \ln 5x$

$$f'(x) = \frac{5}{5x} = \frac{1}{x}$$

12. $f(x) = \ln(3x^2 - 2x + 1)$

$$f'(x) = \frac{6x-2}{3x^2 - 2x + 1}$$

4. $g(x) = \ln(2x+1)$

$$g'(x) = \frac{2}{2x+1}$$

14. $f(x) = \ln(\frac{x+1}{x-1})$

way 1: $f(x) = \ln(x+1) - \ln(x-1)$

way 2: $f'(x) = \frac{1}{x+1} - \frac{1}{x-1}$

$$\begin{aligned} f'(x) &= \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{-2}{(x-1)^2} \cdot \frac{x-1}{x+1} \\ &= \frac{-2}{(x-1)(x+1)} \end{aligned}$$

34. $g(x) = \ln(e^x + \ln x)$

$$g'(x) = \frac{e^x + \frac{1}{x}}{e^x + \ln x} \quad \text{OR} \quad \frac{xe^x + 1}{xe^x + x \ln x}$$

45. $y = \frac{(2x^2-1)^5}{\sqrt{x+1}}$

$$\ln y = \ln \left[\frac{(2x^2-1)^5}{\sqrt{x+1}} \right]$$

$$\ln y = 5 \ln(2x^2-1) - \frac{1}{2} \ln(x+1)$$

$$\frac{y'}{y} = 5 \cdot \frac{4x}{2x^2-1} - \frac{1}{2} \cdot \frac{1}{x+1} = \frac{20x}{2x^2-1} - \frac{1}{2(x+1)}$$

$$y' = \left[\frac{20x}{2x^2-1} - \frac{1}{2(x+1)} \right] \cdot \frac{(2x^2-1)^5}{\sqrt{x+1}}$$

51. $\ln y - x \ln x = -1$

$$\ln y = x \ln x - 1$$

$$\frac{y'}{y} = \ln x + 1$$

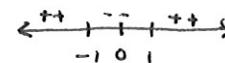
$$y' = (\ln x + 1)y$$

57. $f(x) = x^2 + \ln x^2$ find concavity

$$f'(x) = 2x + 2/x$$

$$f''(x) = 2 - 2/x^2 = 0$$

$$x^2 = 1 \Rightarrow x = \pm 1$$



CU: $(-\infty, -1) \cup (1, \infty)$

CD: $(-1, 0) \cup (0, 1)$

$f'(100) = .0579898266$

$f'(500) = .0139989486$