hw solutions
section 5.6

6. purchased 3 yrs ago for \$500,000
present value: \$320,000
value in 4 ~~yrs~~ yrs ?

$$P(t) = 500{,}000e^{kt}$$
$$P(3) = 320{,}000 = 500{,}000e^{3k}$$
$$\frac{16}{25} = e^{3k}$$
$$\frac{1}{3}\ln(16/25) = k$$
$$-.14876 = k$$
$$P(t) = 500{,}000e^{-.14876t}$$
$$P(7) = 176491.2898$$
or \$176,491.29

9. half-life of 14.2 days
initially 100g of P-32
find amount after t days
after 7.1 days? how fast decaying at $t=7.1$?

$$P(t) = P_0 e^{kt}, \quad P_0 = 100$$
$$50 = 100e^{k(14.2)}$$
$$\frac{1}{2} = e^{14.2k}$$
$$-\ln 2 = 14.2k, \quad k = -.048813$$
$$P(t) = 100e^{-.048813t}$$
$$P(7.1) = 70.71067812$$
$$P'(7.1) = -3.451613$$

20. $$Q(t) = \frac{1000}{1+199e^{-.8t}}$$

(a) $$Q(1) = \frac{1000}{1+199e^{-.8}} = 11.0599$$

(b) $$Q(10) = \frac{1000}{1+199e^{-8}} = 937.42$$

(c) $$\lim_{t\to\infty} \frac{1000}{1+199e^{-.8t}} = \frac{1000}{1+199(0)} = 1000$$

29. $$C(t) = \frac{k}{b-a}\left(e^{-at} - e^{-bt}\right), \quad b > a$$

(a) find t, where $C(t)$ max

$$C'(t) = \frac{k}{b-a}\left(-ae^{-at} + be^{-bt}\right) = 0$$
$$-ae^{-at} + be^{-bt} = 0$$
$$be^{-bt} = ae^{-at}$$
$$\ln(be^{-bt}) = \ln(ae^{-at})$$
$$\ln b - bt = \ln a - at$$
$$\ln b - \ln a = bt - at = (b-a)t$$
$$t = \frac{\ln b - \ln a}{b-a}$$

(b) $$\lim_{t\to\infty} \frac{k}{b-a}\left(e^{-at} - e^{-bt}\right)$$
$$= \frac{k}{b-a}\left[\lim_{t\to\infty} e^{-at} - \lim_{t\to\infty} e^{-bt}\right]$$
$$= \frac{k}{b-a}(0-0) = 0$$

Sect 5.6
skip:
#29b