hw solutions
section 6.2

2. $\int 4x(2x^2+1)^7\,dx$

$u = 2x^2+1$
$du = 4x\,dx$

$= \int u^7\,du = \frac{1}{8}u^8 + C$

$= \frac{1}{8}(2x^2+1)^8 + C$

7. $\int 3t^2\sqrt{t^3+2}\,dt$

$u = t^3+2$
$du = 3t^2\,dt$

$= \int \sqrt{u}\,du = \int u^{1/2}du = \frac{2}{3}u^{3/2} + C$

$= \frac{2}{3}(t^3+2)^{3/2} + C$

17. $\int \frac{2x^2+1}{.2x^3+.3x}\,dx$

$u = .2x^3+.3x$
$du = .6x^2+.3\,dx$
$du = .3(2x^2+1)dx$
$\frac{10}{3}du = (2x^2+1)dx$

$= \frac{10}{3}\int \frac{1}{u}\,du = \frac{10}{3}\ln u + C$

$= \frac{10}{3}\ln(.2x^3+.3x) + C$

$\int \frac{x}{3x^2-1}\,dx$

$u = 3x^2-1$
$du = 6x\,dx$
$\frac{1}{6}du = x\,dx$

$= \frac{1}{6}\int \frac{1}{u}\,du$

$= \frac{1}{6}\ln|u| + C$

$= \frac{1}{6}\ln|3x^2-1| + C$

22. $\int e^{2t+3}\,dt$

$u = 2t+3$
$du = 2\,dt$
$\frac{1}{2}du = dt$

$= \int \frac{1}{2}e^u\,du = \frac{1}{2}e^u + C$

$= \frac{1}{2}e^{2t+3} + C$

37. $\int \frac{1}{x\ln x}\,dx$

$u = \ln x$
$du = \frac{1}{x}dx$

$= \int \frac{1}{u}du = \ln u + C$

$= \ln(\ln x) + C$

42. $\int \left(xe^{-x^2} + \frac{e^x}{e^x+3}\right)dx$

$u = -x^2$ $\quad$ $u = e^x+3$
$du = -2x\,dx$ $\quad$ $du = e^x\,dx$

$= \int xe^{-x^2}dx + \int \frac{e^x}{e^x+3}dx = -\frac{1}{2}\int e^u du + \int \frac{1}{u}du$

$= -\frac{1}{2}e^u + \ln u + C = -\frac{1}{2}e^{-x^2} + \ln(e^x+3) + C$

45. $\int x(x-1)^5\,dx$

$u = x-1$
$du = dx$
$x = u+1$

$= \int (u+1)u^5\,du = \int u^6+u^5\,du$

$= \frac{1}{7}u^7 + \frac{1}{6}u^6 + C = \frac{1}{7}(x-1)^7 + \frac{1}{6}(x-1)^6 + C$

OR $= (x-1)^6\left[\frac{1}{7}x - \frac{1}{7} + \frac{1}{6}\right] = (x-1)^6\left(\frac{1}{7}x + \frac{1}{42}\right)$

52. $f'(x) = \frac{3x^2}{2\sqrt{x^3-1}}$ and $f(1) = 1$

$f(x) = \int \frac{3x^2}{2\sqrt{x^3-1}}\,dx = \int \frac{1}{2\sqrt{u}}du = \sqrt{u} + C = \sqrt{x^3-1} + C$

$f(1) = \sqrt{1-1} + C = C = 1$

$f(x) = \sqrt{x^3-1} + 1$

b/c $u = x^3-1$
$du = 3x^2\,dx$
and $\frac{d}{du}\sqrt{u} = \frac{1}{2\sqrt{u}}$

60. $r(t) = 400\left(1 + \frac{2t}{24+t^2}\right)$, $0 \le t \le 5$

find $P(t) = \int r(t)\,dt$, $P(0) = 60{,}000$

$P(t) = \int 400\left(1 + \frac{2t}{24+t^2}\right)dt$

$= 400\left(t + \ln(24+t^2)\right) + C$

$P(0) = 400\ln(24) + C = 60{,}000$, $C = 58728.77$

$P(t) = 400t + 400\ln(24+t^2) + 58728.77847$

$P(5) = 62285.50659$

61. $r'(t) = \frac{30}{\sqrt{2t+4}}$. find $r(t)$ & $A(t) = \pi r^2$, $t=16$, $r(0)=0$

$r(t) = \int \frac{30}{\sqrt{2t+4}}dt = \int \frac{15}{\sqrt{u}}du$ $\quad$ $u = 2t+4$, $du = 2dt$

$= 30\sqrt{u} + C = 30\sqrt{2t+4} + C$

$r(0) = 30\sqrt{4} + C = 60 + C = 0$, $C = -60$

$r(t) = 30\sqrt{2t+4} - 60$

$r(16) = 30\sqrt{32+4} - 60 = 30\cdot 6 - 60 = 120$

$A(16) = \pi(120)^2 = 14400\pi\ \text{ft}^2$