hw solutions
section 6.5

2. $\int_0^1 x^2 (2x^3-1)^4\,dx$

$u = 2x^3-1$
$du = 6x^2\,dx$
$\frac{1}{6}du = x^2\,dx$

$x \longrightarrow u$
$0 \longrightarrow -1$
$1 \longrightarrow 1$

$= \int_{-1}^{1} \frac{1}{6} u^4\,du = \frac{1}{6}\cdot\frac{1}{5}u^5\Big|_{-1}^{1}$

$= \left(\frac{1}{30}\right) - \left(-\frac{1}{30}\right) = \frac{2}{30} = \frac{1}{15}$

24. $\int_0^1 \frac{e^x}{1+e^x}\,dx$

$u = 1+e^x$
$du = e^x\,dx$

$x \longrightarrow u$
$1 \longrightarrow 1+e$
$0 \longrightarrow 1+1=2$

$= \int_2^{1+e} \frac{1}{u}\,du = \ln u\Big|_2^{1+e}$

$= \ln(1+e) - \ln(2)$

27. $\int_1^2 \left(2e^{-4x} - \frac{1}{x^2}\right)dx$

$= -\frac{1}{2}e^{-4x} + 1/x\,\Big|_1^2$

$= \left(-\frac{1}{2}e^{-8} + \frac{1}{2}\right) - \left(-\frac{1}{2}e^{-4} + 1\right)$

$= -\frac{1}{2}\left(e^{-8} - e^{-4} + 1\right)$

33. $f(x) = e^{-x/2}$, $[-1,2]$. find the area under the region

$\int_{-1}^{2} e^{-x/2}\,dx = -2e^{-x/2}\Big|_{-1}^{2}$

$= -2e^{-1} - (-2e^{1/2}) = -2e^{-1} + 2e^{1/2}$

44. $f(x) = \frac{1}{1+x}$; $[0,2]$. find the avg value

$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2-0}\int_0^2 \frac{1}{1+x}\,dx$

$= \frac{1}{2}\cdot \ln(1+x)\Big|_0^2 = \frac{1}{2}\ln(3) - \frac{1}{2}\ln 1$

$= \frac{1}{2}\ln 3$

45. $P(0) = 3.5$, $P'(t) = 3.5e^{.05t}$
find $P(20) - P(0)$.

$P(20) - P(0) = \int_0^{20} 3.5e^{.05t}\,dt$

$= 70e^{.05t}\Big|_0^{20}$

$= 70e^1 - 70e^0 = 70(e-1)$

$= 120.279728$ bill. metric tons of coal

51. $S(t) = .86t^{.96}$, $1 \le t \le 7$
find S_{Avg} b/t $[1,7]$.

$S_{Avg} = \frac{1}{7-1}\int_1^7 .86t^{.96}\,dt$

$= \frac{1}{6}\left(\frac{.86}{1.96}t^{1.96}\right)\Big|_1^7$

$= \frac{1}{6}\cdot\frac{.86}{1.96}\left(7^{1.96} - 1\right)$

$= 3.24186888$

71. evaluate: $\int_3^3 (1+\sqrt{x})e^{-x}\,dx = 0$

b/c $a=b=3$

72. evaluate: $\int_3^0 f(x)\,dx$
given that $\int_0^3 f(x)\,dx = 4$

$\int_3^0 f(x)\,dx = -\int_0^3 f(x)\,dx = -4$

74. $\int_{-1}^{2} f(x)\,dx = 2$ & $\int_0^2 f(x)\,dx = 3$

(a) $\int_{-1}^{0} f(x)\,dx = \int_{-1}^{2} f(x)\,dx - \int_0^2 f(x)\,dx$

$= 2 - 3 = -1$

(b) $\int_0^2 f(x)\,dx - \int_{-1}^{0} f(x)\,dx$

$= 3 - (-1)$

$= 4$