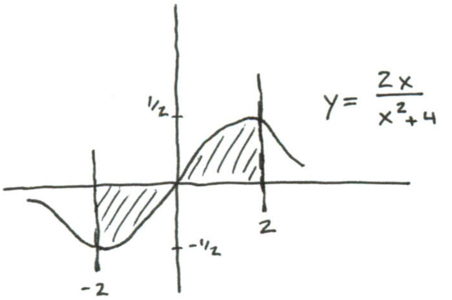
hw solutions
section 6.6

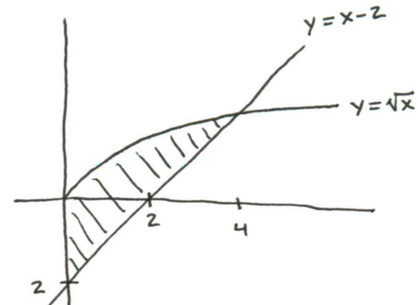
4.



find the area that's shaded.

$$\text{Area} = \int_{-2}^{0} 0 - \frac{2x}{x^2+4}\,dx + \int_0^2 \frac{2x}{x^2+4}\,dx$$
$$= -\int_{-2}^{0} \frac{2x}{x^2+4}\,dx + \int_0^2 \frac{2x}{x^2+4}\,dx$$
$$= 2\int_0^2 \frac{2x}{x^2+4}\,dx$$
$$= 2\ln(x^2+4)\Big|_0^2$$
$$= 2\ln 8 - 2\ln 4$$
$$= 6\ln 2 - 4\ln 2$$
$$= 2\ln 2$$

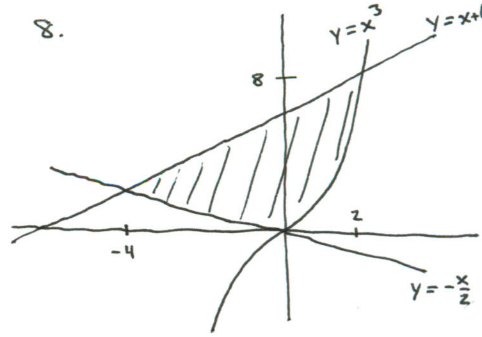
6.



find the area of the shaded region

$$\text{Area} = \int_0^4 \sqrt{x} - (x-2)\,dx$$
$$= \tfrac{2}{3}x^{3/2} - \tfrac{1}{2}x^2 + 2x\Big|_0^4$$
$$= \tfrac{2}{3}\cdot 4^{3/2} - \tfrac{1}{2}\cdot 16 + 8$$
$$= \tfrac{16}{3} - 8 + 8$$
$$= 16/3$$

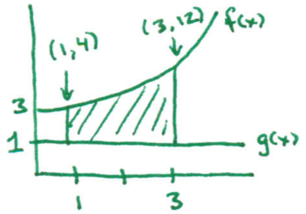
8.



find the area

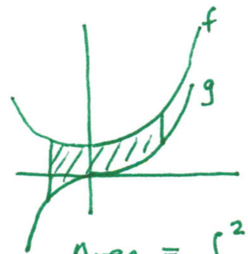
$$\text{Area} = \int_{-4}^{0} (x+6) - \left(-\tfrac{x}{2}\right)dx + \int_0^2 (x+6) - x^3\,dx$$
$$= \int_{-4}^{0} \tfrac{3}{2}x + 6\,dx + \int_0^2 x+6-x^3\,dx$$
$$= \tfrac{3}{4}x^2 + 6x\Big|_{-4}^{0} + \tfrac{1}{2}x^2 + 6x - \tfrac{1}{4}x^4\Big|_0^2$$
$$= -(12-24) + (2+12-4)$$
$$= 12 + 10$$
$$= 22$$

17. $f(x) = x^2+3$, $g(x) = 1$; $a=1, b=3$



$$\text{Area} = \int_1^3 x^2+3-1\,dx$$
$$= \int_1^3 x^2+2\,dx$$
$$= \tfrac{1}{3}x^3\Big|_1^3 = 9 - \tfrac{1}{3} = \tfrac{26}{3} = 8.\bar{6}$$

21. $f(x) = x^2+1$, $g(x) = \frac{1}{3}x^3$; $a=-1, b=2$

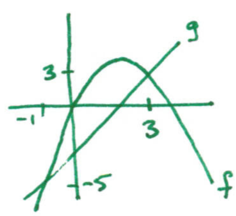


$$\text{Area} = \int_{-1}^{2} (x^2+1) - \tfrac{1}{3}x^3\,dx$$
$$= \tfrac{1}{3}x^3 + x - \tfrac{1}{12}x^4\Big|_{-1}^{2}$$
$$= \left(\tfrac{8}{3} + 2 - \tfrac{16}{12}\right) - \left(-\tfrac{1}{3} - 1 - \tfrac{1}{12}\right) = 4.75$$

23. $f(x) = \frac{1}{x}$, $g(x) = 2x-1$; $a=1, b=4$

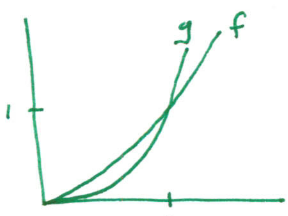
$$\text{Area} = \int_1^4 2x - 1 - \tfrac{1}{x}\,dx$$
$$= x^2 - x - \ln x\Big|_1^4$$
$$= 16 - 4 - \ln 4 - (1-1)$$
$$= 12 - 2\ln 2$$

36. $f(x) = -x^2+4x$ & $g(x) = 2x-3$



$$\text{Area} = \int_{-1}^{3} (-x^2+4x) - (2x-3)\,dx$$
$$= \int_{-1}^{3} -x^2 + 2x + 3\,dx$$
$$= -\tfrac{1}{3}x^3 + x^2 + 3x\Big|_{-1}^{3}$$
$$= 9 - \left(-\tfrac{5}{3}\right) = \tfrac{32}{3}$$

37. $f(x) = x^2$ & $g(x) = x^3$



$$\text{Area} = \int_0^1 x^2 - x^3\,dx$$
$$= \tfrac{1}{3}x^3 - \tfrac{1}{4}x^4\Big|_0^1$$
$$= \left(\tfrac{1}{3} - \tfrac{1}{4}\right) - 0$$
$$= \tfrac{1}{12}$$

43. see p461

$$S = \int_0^b g(x) - f(x)\,dx$$

= the additional revenue the company could get by switching ad agencies.