Homework Set 10

Limit Rules

(sect 1.4, 1.6, and 3.1)

Use Limit Laws to compute the following limits.

1.

$$\lim_{x\to-1}(x^2+1)(3x^4-2x-7)$$

$$= \lim_{x\to-1}(x^2+1)\cdot\lim_{x\to-1}(3x^4-2x-7)$$

$$= \left[\lim_{x\to-1}x^2+\lim_{x\to-1}1\right]\left[\lim_{x\to-1}3x^4-\lim_{x\to-1}2x-\lim 7\right]$$

$$= \left[\left(\lim_{x\to-1}x\right)^2+1\right]\left[3\lim_{x\to-1}x^4-2\lim_{x\to-1}x-7\right]$$

$$= \left[(-1)^2+1\right]\left[3\left(\lim_{x\to-1}x\right)^4-2(-1)-7\right] = 2\left(3(-1)^4-5\right) = -4$$

2.

$$\lim_{x\to0}\frac{e^x+\cos x}{x^2-3x+1}$$

$$= \frac{\lim_{x\to0}(e^x+\cos x)}{\lim_{x\to0}(x^2-3x+1)}$$

$$= \frac{\lim_{x\to0}e^x+\lim_{x\to0}\cos x}{\lim_{x\to0}x^2-\lim_{x\to0}3x+\lim_{x\to0}1}$$

$$= \frac{e^0+\cos 0}{\left(\lim_{x\to0}x\right)^2-3\lim_{x\to0}x+1}$$

$$= \frac{1+1}{1} = 2$$

Use an appropriate method to compute the following limits.

3.

$$\lim_{x\to0}x\sin x = 0\cdot\sin 0 = 0$$

4.

$$\lim_{x\to2}e^{x^2-3x} = e^{2^2-3\cdot2} = e^{-2}$$

5.

$$\lim_{x\to2^-}\frac{1}{x-2} = -\infty$$

6.

$$\lim_{x\to0}\frac{x-5}{x^3-5x^2} = \lim_{x\to0}\frac{1}{x^2} = \infty$$

Use simplification to compute the following limits.

7.

$$\lim_{x\to-1} \frac{x^2+x}{(x+1)(x-3)} = \lim_{x\to-1} \frac{x\cancel{(x+1)}}{\cancel{(x+1)}(x-3)} = \frac{-1}{-4} = 1/4$$

8.

$$\lim_{x\to0} \frac{(2+x)^3-8}{x} = \lim_{x\to0} \frac{12x+6x^2+x^3}{x} = \lim_{x\to0} (12+6x+x^2) = 12$$

$$\text{b/c } (2+x)^3 = (2+x)(4+4x+x^2) = 8+12x+6x^2+x^3$$

9.

$$\lim_{x\to3} \frac{x^2-9}{x^3-x^2-6x} = \lim_{x\to3} \frac{(x+3)\cancel{(x-3)}}{x(x+2)\cancel{(x-3)}} = \frac{6}{3(5)} = \frac{2}{5}$$

10.

$$\lim_{x\to7} \frac{\left(\frac{1}{x}-\frac{1}{7}\right)\cdot 7x}{(x-7)\cdot 7x} = \lim_{x\to7} \frac{7-x}{(x-7)\cdot 7x}$$

$$= \lim_{x\to7} \frac{-\cancel{(x-7)}}{7x\cancel{(x-7)}} = \frac{-1}{49}$$

11.

$$\lim_{x\to-4} \frac{\sqrt{x^2+9}-5}{x+4} \cdot \frac{\sqrt{x^2+9}+5}{\sqrt{x^2+9}+5} = \lim_{x\to-4} \frac{(x^2+9)-25}{(x+4)\left[\sqrt{x^2+9}+5\right]}$$

$$= \lim_{x\to-4} \frac{x^2-16}{(x+4)\left[\sqrt{x^2+9}+5\right]} = \lim_{x\to-4} \frac{x-4}{\sqrt{x^2+9}+5}$$

$$= \frac{-8}{\sqrt{16+9}+5} = \frac{-8}{10} = -4/5$$

12.

$$\lim_{x\to\infty} \frac{x^4 - 5x^2 + 2}{3x^4 + x^3 - 2x} \cdot \frac{\frac{1}{x^4}}{1/x^4} = \lim_{x\to\infty} \frac{1 - 5/x^2 + 2/x^4}{3 + 1/x - 2/x^3}$$

$$= \frac{1 - 0 + 0}{3 + 0 - 0} = 1/3$$

13.

$$\lim_{x\to-\infty} \frac{x^5 + 3x^4 - 7x}{2x^4 + x^2 + x - 1} \cdot \frac{1/x^4}{1/x^4} = \lim_{x\to-\infty} \frac{x + 3 - 7/x^3}{2 + 1/x^2 + 1/x^3 - 1/x^4}$$

$$= \frac{\left(\lim_{x\to-\infty} x\right) + 3 - 0}{2 + 0 + 0 - 0} = \frac{-\infty + 3}{2} = -\infty$$

Use the Squeeze Theorem to compute the following limits.

14. Suppose $3x \leq f(x) \leq x^4 - x + 3$. Compute $\lim_{x\to1} f(x)$.

$$\lim_{x\to1} (3x) \leq \lim_{x\to1} f(x) \leq \lim_{x\to1} (x^4 - x + 3)$$

$$3 \leq \lim_{x\to1} f(x) \leq 3$$

$\therefore \lim_{x\to1} f(x) = 3$ by the squeeze theorem

15.

$$\lim_{x\to1} (x^2 - \sqrt{x}) \cos\left(\frac{1}{x-1}\right)$$

$$-1 \leq \cos\left(\frac{1}{x-1}\right) \leq 1$$

$$-(x^2 - \sqrt{x}) \leq (x^2 - \sqrt{x})\cos\left(\frac{1}{x-1}\right) \leq x^2 - \sqrt{x}$$

$$\lim_{x\to1} -(x^2 - \sqrt{x}) \leq \lim_{x\to1} (x^2 - \sqrt{x})\cos\left(\frac{1}{x-1}\right) \leq \lim_{x\to1} (x^2 - \sqrt{x})$$

$$0 \leq \lim_{x\to1} (x^2 - \sqrt{x})\cos\left(\frac{1}{x-1}\right) \leq 0$$

$\therefore \lim_{x\to1} (x^2 - \sqrt{x})\cos\left(\frac{1}{x-1}\right)$ by the squeeze theorem