Homework Set 12

3.7: L'Hospital's Rule

If the following limits are in the correct form, use L'Hospital's Rule to compute the limit.

1.

$$\lim_{x\to\infty} \frac{e^x}{x^2-1} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{e^x}{2x} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{e^x}{2} = \infty$$

2.

$$\lim_{x\to 0} \frac{\sin 6t}{\tan 3t} \overset{0/0}{=} \lim_{x\to 0} \frac{6\cos 6t}{3\sec^2 3t} = \frac{6\cos(0)}{3\sec^2(0)} = 6/3 = 2$$

3.

$$\lim_{x\to 0} \frac{6^x - 4^x}{x} \overset{0/0}{=} \lim_{x\to 0} \frac{6^x \ln 6 - 4^x \ln 4}{1} = \ln 6 - \ln 4 = \ln(6/4) = \ln(3/2)$$

4.

$$\lim_{x\to\infty} \frac{x^3 - 2x^2 + 1}{4x^2 - 3x + 5} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{3x^2 - 4x}{8x-3} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{6x-4}{8} = \infty$$

5.

$$\lim_{x\to 0^+} x^3 \ln(x) \overset{0\cdot\infty}{=} \lim_{x\to 0^+} \frac{\ln x}{x^{-3}} \overset{\infty/\infty}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-3x^{-4}} = \lim_{x\to 0^+} \frac{x^3}{-3} = 0$$

6.

$$\lim_{x\to 0} (\csc x - \cot x) \quad \overset{\infty-\infty}{}$$

$$= \lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right) = \lim_{x\to 0} \left(\frac{1-\cos x}{\sin x}\right) \overset{0/0}{}$$

$$= \lim_{x\to 0} \frac{\sin x}{\cos x} = \frac{0}{1} = 0$$

7.

$$\lim_{x\to\infty}(x-\ln x) \overset{\infty-\infty}{=} \lim_{x\to\infty} x\left(1-\frac{\ln x}{x}\right) = \left(\lim_{x\to\infty} x\right)\left(1-\lim_{x\to\infty}\frac{\ln x}{x}\right)$$

$$= \left(\lim_{x\to\infty} x\right)\left(1-\lim_{x\to\infty}\frac{\left(\frac{1}{x}\right)}{1}\right) = \left(\lim_{x\to\infty} x\right)(1-0)$$

$$= \lim_{x\to\infty} x = \infty$$

8.

$$\lim_{x\to0^+} x^{\sin x} = e^0 = 1$$

b/c $\displaystyle \lim_{x\to0^+} \ln x^{\sin x} = \lim_{x\to0^+} \sin x\cdot\ln x \overset{0\cdot\infty}{=} \lim_{x\to0^+}\frac{\ln x}{\csc x} \overset{\infty/\infty}{}$

$$= \lim_{x\to0^+}\frac{\frac{1}{x}}{-\csc x\cot x} = \lim_{x\to0^+} -\frac{\sin x\tan x}{x} \overset{0/0}{=} \lim_{x\to0^+} -\frac{\cos x\tan x+\sin x\sec^2 x}{1}$$

$$= 0$$

9.

$$\lim_{x\to\infty}\left(1-\frac{3}{x}\right)^{5x} = e^{-15}$$

b/c $\displaystyle \lim_{x\to\infty} \ln\left(1-\tfrac{3}{x}\right)^{5x} = \lim_{x\to\infty} 5x\ln\left(1-\tfrac{3}{x}\right) = 5\lim_{x\to\infty}\frac{\ln\left(1-\frac{3}{x}\right)}{\frac{1}{x}}$

$$= 5\lim_{x\to\infty}\frac{\frac{3/x^2}{(1-3/x)}}{-1/x^2} = 5\lim_{x\to\infty}\frac{-3}{(1-3/x)}$$

$$= 5(-3) = -15$$

10.

$$\lim_{x\to0^+}(1+4x)^{1/x} = e^4$$

b/c $\displaystyle \lim_{x\to0^+} \ln(1+4x)^{1/x} = \lim_{x\to0^+}\frac{1}{x}\cdot\ln(1+4x) = \lim_{x\to0^+}\frac{\ln(1+4x)}{x}$

$$= \lim_{x\to0^+}\frac{\frac{4}{(1+4x)}}{1} = 4$$