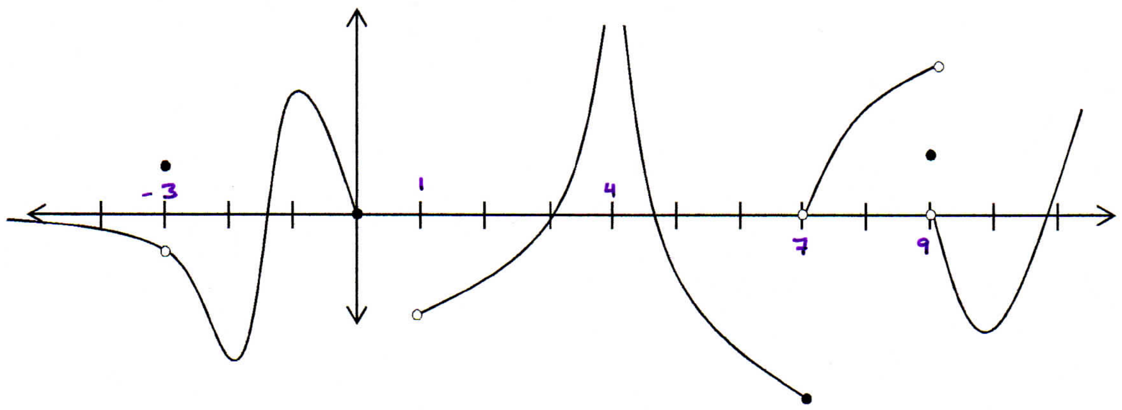
Key

Homework Set 13

1.5: Continuity

1. On what intervals is the function f, shown below, continuous?



$(-\infty,-3)\cup(-3,0)\cup(1,4)\cup(4,7)\cup(7,9)\cup(9,\infty)$

OR $(-\infty,-3)\cup(-3,0]\cup(1,4)\cup(4,7]\cup(7,9)\cup(9,\infty)$

For questions 2 and 3, find where the given function is continuous.

2. $f(x) = \frac{x}{x^2+5x+6} = \frac{x}{(x+2)(x+3)}$

f is continuous when $x \neq -2,-3$

OR $(-\infty,-2)\cup(-2,-3)\cup(-3,\infty)$

3. $g(x) = x^2 + \sqrt{2x-1}$

$2x-1 \geq 0$
$2x \geq 1$
$x \geq 1/2$

g is continuous when $x \geq 1/2$

OR $[1/2,\infty)$

4. Find the numbers at which the function f is discontinuous. At which of these points is the function f continuous from the right, from the left, or neither?

$$f(x) = \begin{cases} x+2 & \text{if } x<0 \\ e^x & \text{if } 0 \leq x \leq 1 \\ (2-x)^2 & \text{if } x>1 \end{cases}$$

<u>at $x=0$</u>:

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0}(x+2) = 2$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} e^x = 1$

So, f is discontinuous at $x=0$; it is right continuous since $f(0)=1$.

<u>at $x=1$</u>:

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} e^x = e$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1}(2-x)^2 = 1$

So, f is discontinuous at $x=1$; it is left continuous since $f(1)=e$.

For all other values of x, $f(x)$ is continuous.

5. Is the given function discontinuous at a? Why or why not? If it is discontinuous, what type of discontinuity is at $x = a$?

a. $f(x) = \frac{1}{x+2} \quad a = -2$

discontinuous at $a=-2$
b/c there is an asymptote
type: infinite discontinuity

b. $g(x) = \begin{cases} \frac{1}{x+2} & , \text{if } x \neq -2 \\ 1 & , \text{if } x = -2 \end{cases} \quad a = -2$

discontinuous at $a=-2$
b/c there is an asymptote
type: infinite discontinuity

c. $h(t) = \begin{cases} e^t & , \text{if } t < 0 \\ t^2 & , \text{if } t \geq 0 \end{cases} \quad a = 0$

discontinuous at $a=0$
b/c $\lim_{t\to 0^-} h(t) = \lim_{t\to 0} e^t = 1$ & $\lim_{t\to 0^+} h(t) = \lim_{t\to 0} t^2 = 0$

type: jump discontinuity

d. $p(t) = \begin{cases} \frac{x^2-1}{x^2-x} & , \text{if } x \neq 1 \\ 2 & , \text{if } x = 1 \end{cases} \quad a = 1$

continuous at $a=1$
b/c $\lim_{t\to 1} p(t) = \lim_{t\to 1} \frac{t^2-1}{t^2-t} = \lim_{t\to 1} \frac{t+1}{t} = 2$

and $p(1) = 2$

6. Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \leq x < 3 \\ 2x - a + b & \text{if } x \geq 3 \end{cases}$$

need: $\begin{cases} \lim_{x\to 2^-} f(x) = \lim_{x\to 2} \frac{x^2-4}{x-2} = 4 \\ \lim_{x\to 2^+} f(x) = f(2) = 4a-2b+3 \end{cases}$

and $\begin{cases} \lim_{x\to 3^-} f(x) = 9a-3b+3 \\ \lim_{x\to 3^+} f(x) = f(3) = 6-a+b \end{cases}$

So, $\begin{cases} 4a-2b+3=4 \\ 9a-3b+3=6-a+b \end{cases}$

$\Rightarrow \begin{cases} 4a-2b=1 \\ 10a-4b=3 \end{cases}$

Solve for a & b yeilds:

$a = 1/2$
$b = 1/2$

For questions 7 and 8, use continuity to calculate the limits.

7.

$$\lim_{x \to \pi} \sin(x + \sin x)$$

$$= \sin(\pi + \sin(\pi)) = \sin(\pi + 0) = 0$$

8.

$$\lim_{x \to 2} \arctan\left(\frac{x^2 - 4}{3x^2 - 6x}\right)$$

$$= \arctan\left(\lim_{x \to 2} \frac{(x+2)\cancel{(x-2)}}{3x\cancel{(x-2)}}\right)$$

$$= \arctan\left(\frac{4}{6}\right) = \arctan(2/3)$$

For questions 9 and 10, use the Intermediate Value Theorem to show that there is a root of the equation in the given interval.

9. $x^4 + x - 3 = 0 \quad on \ (1,2)$

$f(x) = x^4 + x - 3$

$f(1) = 1 + 1 - 3 = -1$

$f(2) = 16 + 2 - 3 = 15$

by the IVT, there exists $x = c$ such that $f(c) = 0$

10. $\sin x = x^2 - x \quad on \ (1,2)$

$f(x) = \sin x - x^2 + x$

$f(1) = \sin(1) - 1 + 1 = \sin(1) = .8414709848$

$f(2) = \sin(2) - 4 + 2 = \sin(2) - 2 = -1.090702573$

by the IVT, there is a $x = c$ such that $f(c) = 0$